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7/25/68

THE BEARING CAPACITY OF A TWO-LAYERED SOIL SYSTEM

A THESIS

Presented to

The Faculty of the Graduate Division

by

Anthony Thomas Avella

In Partial Fulfillment

of the Requirements for the Degree

Master of Science in Civil Engineering

Georgia Institute of Technology

June, 1969

THE BEARING CAPACITY OF A TWO-LAYERED SOIL SYSTEM

Approved:

Chairman

Date approved by Chairman: 6 June 1969

ACKNOWLEDGEMENTS

The writer wishes to express his gratitude to his thesis advisor, Professor George F. Sowers, and to the members of his reading committee, Dr. B. B. Mazanti and Dr. A. B. Caseman for their encouragement and constructive criticisms during the preparation of this thesis.

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SUMMARY

The purpose of this experimental investigation was to determine the relationship between the ultimate bearing capacity of a two-layered soil system and the length of model footings used to transmit the load in the system. Also, the effects of varying the thickness of the upper hard layer were studied. Rigid steel model footings of constant width and thickness and of varying length were used. The soil system consisted of a hard layer (unconfined compressive strength = 200 psi) of finite thickness overlying a very soft layer (unconfined compressive strength = 2.72 psi) considered to be of infinite thickness.

The hard layer consisted of a mixture of plaster-of-paris and water. The soft layer consisted of equal parts of bentonitic and kaolinitic clays mixed with water.

Static load tests were conducted on this system to determine the ultimate bearing capacity. The footings used were one inch wide and varied in length as follows: four inches, eight inches and 22 inches. To study the effects of the thickness of the upper hard layer on the ultimate bearing capacity of the system, tests were conducted on slabs[1] that were 1/4 inch, 1/2 inch, 3/4 inch and

1. The terms "upper hard layer" and "slab" are used interchangably in this report.

one inch thick placed over the clay layer. The slabs were 22 inches square in plan view. The ultimate bearing capacity of the system was taken as that pressure at which failure, or sudden sinking of the footings took place.

It was found that the ultimate bearing capacity of this two-layered system is related to the ratio, L/t, where L is the length of the footing and t is the thickness of the hard layer. It was also found that the ultimate bearing capacity was related to the L/B ratio, where L is the length of the footing and B is the length of the slab. It was further determined that for a given footing length the ultimate bearing capacity of the system increased with an increase in thickness of the upper layer. Also, for a given upper layer thickness, the ultimate bearing capacity of the system decreased with an increase in footing length.

Two distinct modes of failure were found. These appear to be related to the thickness of the upper layer and the length of the footing involved. The two types of failures found to occur were a punching shear type failure and a flexural failure.

The punching type failure occurred with the four and eight inch footings while the flexural type failure occurred with the 22 inch footing. It appears that, for a given slab thickness, as the L/t or L/B ratio increases the type of failure goes from a punching to a flexural failure. The same type of failure occurred no matter what thickness of slab was being used.

In general, for the footings with lengths of four and eight inches, there was a sudden punching shear failure that occurred for all slab thicknesses. Immediately preceding this punching type failure, cracks developed from the corners of all of the footings spreading out at approximately a 45° angle. As the footing punched through the upper layer, bulges occurred on each side along the long dimension of the footing indicating shear in the soft layer. These bulges force the broken slab upward adjacent to the footing. The soft layer appeared to be completely remolded beneath and adjacent to the footing.

For the footing 22 inches in length, a flexural type failure occurred when it was placed over the upper layer and loaded to failure. As the footing was being loaded, cracks developed in a longitudinal direction adjacent to the footing. As the ultimate bearing capacity of the system was reached, the upper layer failed in flexure thus causing the footing to push through into the softer layer below. Bulges occurred on each side of the footing and the slab on each side was pushed upward by bulges occurring in the softer layer below.

In general, for a given footing length, there exists an almost straight-line relationship between the ultimate bearing capacity of the system and the upper layer thickness. For a given footing size, as the upper layer thickness is increased, the load required to produce failure of the system is also increased.

For a given upper layer thickness, there exists a relationship between the ultimate bearing capacity of the system and the length of the footing. This relationship is nonlinear. In general, for a given slab thickness, as the length of the footing increases the ultimate bearing capacity of the system decreases.

CHAPTER I

INTRODUCTION

In dealing with foundation design, two main criteria must be met in all circumstances. First, the soil beneath the footing must not fail and, second, the total settlement of the foundation must be kept within the limits that can be tolerated by the particular structure involved. In this experiment, we were only concerned with the ultimate bearing capacity of the system and not with settlement requirements.

When footings are founded on the surface of a hard layer which overlies a soft layer, the pressure applied by the footing spreads out with increasing depth. Therefore, the pressures reaching the soft stratum depend on the thickness of the upper hard stratum as well as on its ability to spread the load.

This two-layered phenomena can occur in nature as dense deposits of sand overlying soft clay layers, as in the Coastal Plain regions; as solutioning in limestone deposits; or as ledges of resistant rock underlain by highly weathered material. Problems may arise when structures are founded on the hard crust without concern for the soft layer below.

It is apparent that all of the structural features of the hard layer and the soft layer must be known. The number of variables in an experiment of this nature are many (C_1, ϕ, E_1/E_2, etc.). The most relevant facts obtained from an experiment of this nature would be the relationships between the ultimate bearing capacity of the system, the thickness of the upper hard layer, the lengths of the footings used to transmit the load to the system and the L/t and L/B ratios.

Purpose of Research

Engineers have been designing footings for decades. Their analysis is based on laboratory experimentation, theory and in-situ testing. The design of footings involved in two-layered systems has been overlooked in the past mainly because of a lack of research; research that will necessarily show the relationships between the variables of the two-layered systems and the ultimate bearing capacity and settlement characteristics of the system. Only after more investigations of this nature will more confidence be shown in the design of footings bearing on a hard crust overlying a very soft stratum. This type of research provides the essential link between theory and practice.

The object of this experiment was to see if there was a relationship between the thickness of the upper layer in a two-layered system and the length of the footings utilized to transfer the load to the two-layered system. This and other similar experiments would necessarily aid in the development of bearing capacity formulas for two-layered systems. The upper hard crust of this system consisted of a mixture of plaster-of-paris and water while the lower, weaker layer consisted of equal parts of bentonite and kaolinite clays mixed with water. The difference in the moduli of elasticity of these two materials was extreme and the ratio of the cohesion of the lower layer to the apparent cohesion of the upper layer approached zero. Because of this fact, these test results can be compared with those obtained through the analyses and tests of Button (Reference 10).

Ultimate Bearing Capacity

The bearing capacity of a soil is defined as the maximum load per unit of area which the soil can support without rupture. Most of the theoretical analyses of ultimate bearing capacity are based on the assumption that the soil is a homogeneous, isotropic and plastic material throughout the zone of soil shear. When the soil is inhomogeneous, these methods are not strictly applicable. The shear pattern is distorted and the area of that portion of the rupture surface in the weaker material will tend to increase while that in the stronger material will decrease (Reference 12).

Homogeneous Soil

The generally accepted formulas for ultimate bearing capacity are listed below (Reference 5). These formulas assume that the footing is resting on the surface of an ideal homogeneous and isotropic material.

Strip Footing

$$q_u = cN_c + 0.5\, B \gamma N_\gamma \qquad (1)$$

Square Footing

$$q_u = 1.3\, cN_c + 0.4\, B \gamma N_\gamma \qquad (2)$$

Circular Footing

$$q_u = 1.3\ cN_c + 0.3\ B\,\gamma\,N_\gamma \tag{3}$$

where

q_u = ultimate bearing capacity

c = cohesion of the material

γ = unit weight of the material

B = linear dimension of footing, width or diameter

ϕ = angle of internal friction of the material

N_c, N_γ = factors depending upon the angle of internal friction of the material

CHAPTER II

TWO-LAYER SYSTEM THEORIES

Burmister Theory of Stresses and Displacements in Layered Systems

D. M. Burmister (Reference 1) was the first to develop a theory of elastic stresses and displacements in a two-layered system. His research was applied to flexible pavement design, especially for airport runways. He investigated a condition of an elastic, homogeneous, isotropic layer of finite thickness H, resting on the surface of a semi-infinite, homogeneous, isotropic mass with the boundary between them sufficiently rough so that no slip would occur. Burmister assumed that the surface reinforcing layer was weightless and infinite in the horizontal direction. He assumed that Hooke's Law was valid for both materials. Finally, he assumed that the two layers were continuously in contact and acted together as an elastic medium of composite nature and that the lower stratum provides initially a continuous uniform support for the top layer. Burmister's analysis showed that the stress at any level is a function of the same factors as in the Boussinesq analysis plus the following:

A. The ratio of the footing width to the thickness of the upper layer.

B. Relations involving the Poisson's ratios of the two strata.

C. The ratio of the moduli of elasticity of the two strata.

In general, based on Burmister's analysis, it can be said that when a rigid layer overlies a soft layer the stresses in the boundary, immediately beneath the load, will be less than those given by the Boussinesq. The rigid layer tends to bridge over the less rigid one and thereby spreads the stress further. According to Burmister, as the radius of the bearing area increases for a given height of the upper layer and for a given E_2/E_1 (the moduli of elasticity of the lower and upper layers, respectively), then the settlement increases and the maximum load that can be applied is, for a given limiting settlement, reduced.

The writer's experiments deviated from the Burmister theory because the Burmister theory assumes that the two materials that comprise the two-layered system function as perfectly elastic materials. This in essence is not the case when dealing with bearing capacity problems that involve the ultimate strength of the materials which are well beyond any elastic range of loading. Burmister, however, brings out the fact of the critical nature of the continuous contact between the upper and lower layers.

McLeod Method

The method of McLeod (Reference 2) includes c (the cohesion) and ϕ (the angle of internal friction) of the two materials and is applied to strip footings on flexible pavements. In his method, McLeod used a logarithmic spiral failure surface. Calculation of the bearing capacity using the log-spiral involves balancing moments about the origin of the spiral. The resultants of the normal forces "n" and the friction forces "n tan ϕ" pass through the origin. The forces tending to cause movement are the applied pressure "p" plus the weight of foundation material to the left of the origin of the spiral. The resisting forces are cohesion, acting along the surface of sliding and the weight of material to the right of the origin within the spiral (Figure 1).

In essence, McLeod's method involves the determination of c and ϕ values for an equivalent homogeneous material having the same ultimate strength as the layered system. The ultimate bearing capacity of this equivalent homogeneous material can then be calculated on the basis of a logarithmic-spiral failure surface. The entire procedure requires a trial and error approach involving the use of successive approximations. Since the ultimate strength of the flexible pavement is employed, which is far beyond any elastic range of loading the structure may have, the method described in the McLeod's paper is based upon the plastic rather than the elastic behavior of the materials and the various layers of the flexible pavement. A comparison with the McLeod method is made a part of this paper.

Meyerhoff Analysis

G. G. Meyerhoff (Reference 8,9) has presented a mathematical approach to the determination of the ultimate failure loads for rigid slabs on an elastic subgrade. An estimate of the ultimate load is obtained from an ultimate strength analysis of slabs on the basis of plastic theory. Meyerhoff uses his method to estimate the ultimate bearing capacity of rigid pavements under concentrated, central, edge and corner loads acting on the pavement. Meyerhoff concerns himself with a central load acting on a circular contact area which was not investigated in the present experiment.

Button Analysis

The ultimate bearing capacity of a long strip footing placed on the surface of a semi-infinite cohesive soil ($\phi = 0$) is a constant multiplied by the cohesion. The values of the factor N_c vary within limits according to the method of analysis used. The cylindrical failure surface proposed by Fellenius for the determination of the stability of a homogeneous soil has been used by Button (Reference 10) to find the bearing capacity of infinitely long footings placed on a two-layer cohesive subsoil. The ultimate bearing capacity is obtained by equating the moment of applied load about the center of the critical circular arc to the reaction moment about the same point. The reaction moment is given by the summation of the total tangential shearing resistance, s, given by the well-known Coulomb equation,

$$s = c + n \tan \phi \qquad (4)$$

multiplied by the radius of the arc. The magnitude of the normal pressure component "n tan ϕ" acting on each element of the circular arc is not definitely known. To avoid this difficulty, Button limited the Fellenius' method to homogeneous cohesive soils for which the angle of internal friction $\phi = 0$, so that n tan ϕ also equals 0. When the cohesions c_1 (upper layer) and c_2 (lower layer) and the thickness d of the upper layer have been determined, it is a simple matter to obtain the value N_c for the value of c_2/c_1 from Button's chart. When c_2/c_1 is less than 1, the lower layer is weaker than the upper one. The value of N_c for any given value of d/b increases as c_2/c_1 increases until it reaches the limiting value of 5.51 and it then remains at this value. At this limiting point, the slip circles lie wholly within the upper layer and any further relative increase in the strength of the lower layer does not increase the bearing capacity. A comparison of the experimental values obtained and the values obtained by the Button analysis are included in this paper.

CHAPTER III

DESCRIPTION OF TEST APPARATUS, MATERIALS AND PROCEDURES

The materials used to construct the two-layered system were plaster-of-paris and water and a mixture of equal parts of bentonite and kaolinite clays and water. The thicknesses of the upper layer used in this experiment were 1/4 inch, 1/2 inch, 3/4 inch and 1 inch. The model footings used had a height and width of one inch and were four, eight and 22 inches long.

Upper (Hard) Layer

The upper layer consisted of a 13 to 10 mixture by weight, of plaster-of-paris and water. To eliminate as many variables as possible, each slab was cast under controlled environmental conditions at the Georgia Tech Ceramics Laboratory and each was allowed to air cure for seven days before testing. The end product turned out to be a relatively hard and brittle material with the properties shown in Table 1.

The values were obtained from cylindrical specimens two inches in diameter by four inches in length. As with the slabs, these specimens were allowed to cure for seven days before they were tested. Unconfined compression and triaxial shear tests using confining pressures of 27.8 psi (4 ksf), 50 psi, 100 psi, 150 psi, and 200 psi were run in a triaxial cell. The modulus of elasticity was determined by taking stress-strain readings during the running of the unconfined test. The modulus of elasticity was also determined

Table 1. Properties of the Plaster-of-Paris Mix

Unconfined Compressive Strength	564 psi
Modulus of Elasticity	58,000 psi
Apparent Cohesion	200 psi
Apparent Angle of Internal Friction	14°
Tensile Strength	240 psi
Wet Density	95 pcf

using a confining pressure of four ksf. It was shown to increase under this confining pressure to 76,000 psi. Tensile specimens were tested in accordance with ASTM:C190.

A wood and plexiglass form, designed by the writer, was used to cast the slabs. The slabs were 22 inches by 22 inches in plan and varied in thickness. The slabs were allowed to harden for seven days prior to testing.

Lower (Soft) Layer

The soft layer consisted of equal parts of bentonite and kaolinite clays mixed with water. The physical properties were as shown in Table 2.

Procedure

The apparatus used for testing the two-layered system was designed by the writer utilizing the facilities of the Georgia Tech Soils Laboratory and the help of the personnel in charge of the laboratory. The test apparatus used in this experiment was relatively simple, easy to assemble and proved to be highly satisfactory. A portable metal bin divided into two compartments was used for housing the clay mass. Each compartment was 24 inches by 24 inches in plan and was 24 inches deep. Only one compartment was used during the testing. A steel I-beam was located over the top of the compartments. Attached to the I-beam and situated directly over the center point of the compartment being used was a ball bearing housing providing a frictionless enclosure for a stainless steel rod (Figure 2).

Table 2. Properties of the Clay Mixture

Water Content	300 per cent
Unconfined Compressive Strength	2.72 psi
Modulus of Elasticity	95 psi
Shear Strength (vane shear)	1.34 psi
Wet Density	77 pcf

Attached to the rod was a platform for balancing the loading blocks. With such an assembly, the load could be applied directly to the center of each footing and perpendicular to the two-layered system. Weights of 5, 10, 20 and 50-pound capacity were utilized to provide the loading for this system. Micrometer dial gauges were used for measuring the deflection at the center of each footing. The gauges were set up under a system that was independent of the loading system.

The clay mixture was hand placed in one of the compartments of the metal bin. The clay was placed in one to two-inch thick lifts and smoothed until a relatively homogeneous mass was obtained. This procedure was followed until the depth of the clay was 24 inches. Since homogeneity of this stratum is a basic requirement in existing two-layer system theories, a considerable amount of time and effort was spent in the preparation of this strata. Homogeneity is assumed in theoretical analysis and an attempt to achieve this requirement should be made part of any research involving two-layered systems.

After the clay stratum had been prepared, the surface was carefully leveled with an aluminum straight edge. This procedure was of great importance in this experiment since contact between the two layers at all points must be insured to be consistent with present theory and also slippage must be prevented by insuring that the vertical force is perpendicular to the soil system. The continuity effect between the layers is one of the major points in all two-layered system studies.

In the writer's opinion, the most critical point during the experiments of this nature is the contact between the upper layer and the lower layer. Should there exist discontinuities or lack of complete contact between the upper and lower layer, changes in the behavior of the system are probable.

The upper stratum was then placed on the lower stratum and a model footing set in the center of the slab. Loads were applied to the system by means of weights and readings of deflection were obtained from the micrometer dial gauges. After failure, the upper stratum was completely removed and the lower stratum was removed to a depth of 12 inches. The clay was then replaced in the manner previously described. The same procedure was followed throughout all the testing. After each failure, the effects on the upper stratum were noted and sketches drawn of each failure surface.

It was noted that the crack pattern on all of the slabs tested followed one of two distinct patterns. On the long footing, the crack pattern was longitudinal, parallel to the length of the footing. For the other footings tested, the crack pattern was diagonal between the four corners of the footing and the four corners of the slab being tested with a shear failure in the center representing the contact area of the footing.

CHAPTER IV

DISCUSSION

The first objective of this investigation was to determine experimentally the relationship between the ultimate bearing capacity of a two-layered system and the length of footings used to transmit loads to the system. The second objective was to determine the effect of the thickness of the hard layer on the ultimate bearing capacity of the system. The two-layer system consisted of a thin, hard upper layer overlying a thick, soft lower layer.

Ultimate Bearing Capacity

The ultimate bearing capacity of the system was defined as the load which produced a total failure of the soil system divided by the area of the footing being used. To compare the ultimate bearing capacity of the system with the bearing capacity of the clay layer, equation (1) was used, based on the results of triaxial tests.

The ultimate bearing capacity for continuous strip footings on clay is given by (Reference 5),

$q_u = 5.14c$ for smooth base footings

where c = cohesion

$q_u = 5.14\ (1.34) = 6.88$ psi

For rectangular footings, a correction factor (Reference 4) must be applied to N_c which depends on the L/b ratio--where L is the length of the footing and b is the width. For the four-inch long footing, the correction factor:- 1.07.

$$q_u = 5.14\ (1.07)\ c$$

$$q_u = 7.37\ \text{psi}$$

One static load test was run using the 22-inch long footing directly in contact with the surface of the clay layer. The ultimate bearing capacity was 5.91 psi, which is within 0.97 psi of the theoretical result at 6.88 psi.

Length of Footing

Figures 3, 4, 5, and 6 show that for a given slab thickness, as the ratio of the length of the footing to the thickness of the slab increases, the ultimate bearing capacity of the system decreases. Figures 7, 8, 9 and 10 show that for a given slab thickness, as the ratio of the length of the footing to the length of the slab increases, the ultimate bearing capacity of the system decreases. The L/t curves are valid for this experiment but may not be universally valid for conditions where the L/B ratios are different. Figures 11, 12, 13 and 14 show that for a given slab thickness, as the length of the footing was increased, the ultimate bearing capacity of the system decreased. Figure 15 shows that the ultimate bearing capacity of the system increases as the ratio of the area of the footing to the thickness of the hard layer decreases. A similar curve would be seen if L/t were plotted as the abscissa since the width of the footings was a constant of one inch. However, this fact may not be true if other footing widths were used. The decrease in bearing capacity with increasing A/t is non-linear. The plotted curve would indicate that the decrease is independent of the relative length to the width of the footing and the thickness of the hard layer, but is dependent upon the ratio of the two.

Mechanism of Failure

In this experiment, two types of failures occurred depending upon the length of the footing being tested. With footings of lengths four inches and eight inches, a punching-type failure occurred. The footing suddenly sank making a hole in the hard layer approximately the same size and shape as that of the footing (Figure 16) and fracturing the hard layer at angles of 45° from the corners of the footing. In this case, it appears that there is a high stress density along the perimeter of the footing which causes the punching-type failure. Even though the upper hard layer does not develop much flexural resistance, it helps to spread the load over the surface of the clay layer. The punching-failure was followed by shear and heave of the clay.

With increasing length of the footing, the hard layer begins to fail in flexure. As the load on the slab increases, the bending stresses below the load become equal to the flexural strength of the plaster-of-paris slab and it begins to yield. As the load is increased, cracks appear on the surface of the slab parallel to the long dimension of the footing (Figures 17 and 18). After the longitudinal cracks occurred, complete failure by punching through the slab into the clay occurred with no increase in load. This punching was followed by shear and heave of the clay, usually on both sides of the footing. In some cases there was heave of the clay only on one side of the footing. This may have been due to poor contact pressure beneath the footing, inhomogeneity of the materials used to make up the two-layered system or lack of complete contact between the two layers. With the footings of lengths four inches and eight inches, the radial cracks always started from the corners of the footings, which would indicate that there is a high concentration of stress at these points. The concentration of stresses at the corner of these footings is due to shear strength of the hard layer. If the shear strength were zero, the uniform settlement of the rigid footing would produce no pressure at the corners due to the lack of resistance to shear at this point. This may not be true for the 22-inch footing, which is as long as the slab, since there are no end effects.

Thickness of Hard Layer

Figures 19, 20 and 21 show that for a given footing length, as the ratio of the length of the footing to the thickness of the upper layer increased, the ultimate bearing capacity of the system decreased. This decrease in ultimate bearing capacity with increases in the L/t ratio is a non-linear function. The L/t curves are valid for this experiment but may not be universally valid for conditions where the L/B ratios are different. Figure 22 shows that for a given L/B ratio, as the thickness of the upper layer increased, the ultimate bearing capacity of the system increased. This increase in the ultimate bearing capacity with increases in the slab thickness appears as a straight-line function. This relationship may not be valid for L/B ratios greater than 1 but probably is a constant. Further, the width of the footing relative to the width of the upper layer may have an effect on the L/B relationship. Also, the relationship may not be correct for very small values of L/B (i.e. as L/B approaches zero). In this instance the bearing capacity of the system probably approaches that of the lower layer. For the L/B values calculated in this experiment, a straight-line relationship was found. Figure 23 shows an almost linear relationship between the slab thickness and the ultimate bearing capacity of the system for a given footing length. As the thickness of the upper layer increased, the bearing capacity of the two-layer system also increased. Regardless of the type of failure, punching or flexural, the ultimate bearing capacity of the system increased with increasing thicknesses of the hard layer.

Theoretical and Test Result Comparison

Table 3 shows the relationship between the ultimate bearing capacity of the two-layer system obtained by experimentation and the allowable bearing pressure as determined by the formula outlined in the NAVDOCKS Design Manual No. 7. The NAVDOCKS manual recommends that where the bearing stratum is underlain by a weaker material the allowable bearing pressure is determined as follows:

$$\frac{Q}{(B + 1.16H)\ (L + 1.16H)} = \text{nominal value of allowable bearing pressure}$$

Table 3. Ultimate Bearing Capacity: Navdocks Formula vs. Experimental Results

Size of Slab (in.)	Length of Footing (in.)	q_o[1] (psi)	q_{all}[2] (psi)
1/4	4	22.5	16.3
1/4	8	16.3	12.2
1/4	22	8.2	6.3
1/2	4	45.0	25.0
1/2	8	31.2	18.4
1/2	22	15.0	9.2
3/4	4	67.5	29.7
3/4	8	45.0	21.7
3/4	22	23.2	11.9
1	4	87.5	31.5
1	8	58.7	23.7
1	22	30.9	13.6

1. The ultimate bearing capacity as obtained by this experiment. (The pressure at failure.)

2. The nominal allowable bearing capacity as computed by the NAVDOCKS formula. (The allowable pressure on the lower layer.)

where Q = applied load, not including weight of foundation itself

B = width of the footing

L = length of the footing

H = thickness of the upper layer below the footing

The computed allowable bearing capacity is about one-half as much as the values obtained from this investigation. This comparison would indicate that the test results give a good correlation with practice.

Table 4 shows the relationship between the ultimate bearing capacity as determined by this experimentation and the bearing capacity as calculated by the Button theory. The Button theory utilizes the following equation,

$$q_u = c_1 N_c$$

where q_u = the ultimate bearing capacity of the system

c_1 = the cohesion of the upper layer

N_c = is a factor based on the angle of internal friction of the material

Since the ratio of the cohesion of the lower layer divided by the cohesion of the upper layer was so small the N_c factors were determined as if c_2/c_1 equaled 0. Having the relationship between c_2 and c_1 established and knowing the d/b factors, one can easily go to Button's chart and determine the N_c factors for each upper layer thickness. The results show that there is a drastic variation between the experimental results and the results as computed by Button's

Table 4. Ultimate Bearing Capacity:
Button Method vs. Experimental Results

Size of Slab (in.)	Length of Footing (in.)	N_c [1]	q_u [2] (psi)	q_u [3] (psi)
1/4	22	0.6	120	8.2
1/2	22	1.1	220	15.0
3/4	22	1.9	380	23.2
1	22	2.4	480	30.9
1/4	8	0.6	120	16.3
1/2	8	1.1	220	31.2
3/4	8	1.9	380	45.0
1	8	2.4	480	58.7
1/4	4	0.6	120	22.5
1/2	4	1.1	220	45.0
3/4	4	1.9	380	67.5
1	4	2.4	480	87.5

1. Factors obtained from Button's chart for $c_2/c_1 = 0$.
2. Ultimate bearing capacity as computed by Button's formula, $q_u = c_1 N_c$.
3. Ultimate bearing capacity as obtained by this experiment.

formula. Button's analysis assumed that $\phi = 0$ for both stratum. As was determined in the triaxial testing, ϕ of the plaster-of-paris mixture was not 0, but was in fact 14°. McLeod has shown in his studies that for slight variations in the ϕ values of materials the ultimate bearing capacity of systems is increased.

We would expect larger values of ultimate bearing capacity to be obtained by this investigation than by the use of Button's formula since the upper layer material had a significant ϕ value. Exactly the opposite occurred. The ultimate bearing capacity as determined by Button's formula gave results that were 15-20 times higher than the results obtained in this investigation. The author concludes that comparison of Button's analysis to this investigation is not exactly valid for the following reasons:

1. Button assumed a shear failure along a circular arc that passed through both layers. In some cases a punching shear failure occurred while in others a flexural failure occurred.

2. Button's equation ($q_u = c_1 N_c$) may not apply to systems that have significant ϕ values.

3. Button's equation may only apply for certain ranges in cohesion values.

4. The rigidity (brittleness) of the upper layer may have a significant effect on the method of analysis used, i.e. as the modulus of elasticity of the upper layer increases, Button's formula may not apply.

5. Button assumed a semi-infinite upper layer in the horizontal direction. This is not the case especially when using the 22-inch

footing. Therefore, edge effects may contribute an important factor in the ultimate strength of the soil system. The smaller footings, four and eight inches are more in line with Button's hypothesis and show results that are in closer agreement to the results computed using Button's formula.

Table 5 shows the relationship between the ultimate bearing capacity as determined by this experimentation and the bearing capacity as calculated by the McLeod theory. In essence, the McLeod method involves the determination of c and ϕ values for an equivalent homogeneous material having the same ultimate strength as the layered system. The ultimate strength of this equivalent homogeneous material is calculated on the basis of a logarithmic spiral failure surface (Figure 1).

When calculating the ultimate strength of a homogeneous soil on the assumption of a logarithmic spiral failure curve, we employ the principal of mechanics that says for equilibrium the sum of the moments of the forces about any point is zero. In this case, it is most convenient to select the origin of the spiral as the point about which the moments are to be taken.

At equilibrium,

load moment = reaction moment = weight moment plus cohesion moment

The load moment is obtained by multiplying the total load by the moment arm. The reaction moment consists of two quantities, the weight moment and the cohesion moment.

Since more material is contained within the spiral to the right than to the left of the vertical through its origin, this unbalanced weight results in a weight moment. If the material under load possesses any cohesion, its cohesion c acts as a shearing resistance along the entire length of the spiral. The summation of the moments for cohesion c for each element of length of the spiral about the spiral's origin gives the cohesion moment.

E. S. Barber (Reference 3) has published several tables of basic data that greatly simplify calculations involving the logarithmic spiral. These enable the weight moment and the cohesion moment for the critical spiral, and the ultimate strength, to be determined quite readily for loads applied to homogeneous soils with different c and ϕ values.

By trial, the position of the critical logarithmic spiral is located such that the shearing resistance of the materials is a minimum for the given footing.

With the assistance of Mr. Donald E. Dixon of Law Engineering Testing Company, Atlanta, Georgia, a computer program was developed for the solution of McLeod's formulas for weight moment and cohesion moment.

The tabulation shows a reasonable correlation between the experimental results and the results as computed by the McLeod formulas. The writer feels that possible reasons for the difference in results are as follows:

1. McLeod assumed a shear failure along a logarithmic spiral that passed through both layers. Two types of failures actually

Table 5. Ultimate Bearing Capacity:
McLeod Method vs. Experimental Results

Size of Slab (in.)	Length of Footing (in.)	q_o [1] (psi)	q_u [2] (psi)
1/4	4	22.5	69.0
1/4	8	16.3	37.5
1/4	22	8.2	18.6
1/2	4	45.0	134.0
1/2	8	31.2	69.1
1/2	22	15.0	29.4
3/4	4	67.5	202.6
3/4	8	45.0	101.2
3/4	22	23.2	40.5
1	4	87.5	281.0
1	8	58.7	134.2
1	22	30.9	51.6

1. The ultimate bearing capacity as obtained by this experiment.

2. The ultimate bearing capacity as determined by McLeod's method.

occurred in this experiment: punching and flexural.

2. McLeod assumes simultaneous mobilization of shear strength in both layers. This, in fact, is not the case.

3. McLeod's method applies to flexible pavement design. It may not be valid when applied to a brittle material as was the fact in this case.

McLeod's theory to the solution of layered systems is more realistic than is Burmister's theory for flexible pavement design. Burmister's layered system theory of flexible pavement design is based upon the elastic properties of the material in each layer. It will result in the same parameters for either cohesionless materials or those containing a binder to give cohesion c, if their moduli of elasticity are the same. The logarithmic spiral method employed by McLeod indicates that while two materials, one cohesionless and the other cohesive with both c and ϕ values might have the same moduli of elasticity, the cohesive material may develop a higher or lower ultimate strength, depending on the relative values of ϕ of the two materials.

The test results appear to indicate that the hard upper stratum has an important reinforcing and load spreading capacity. These are favorable aspects in reducing the stresses and deformation of the soil of layer 2. The system supported loads that were two to ten times or more in excess of those that the bottom layer could support for a given footing.

Further experimenting is needed to see if a bearing capacity formula similar to the general bearing capacity formulas of Meyerhoff or Button can be developed for a two-layered system. To develop this equation, terms involving the moduli of elasticity of the two layers, the cohesion of the two layers and the angle of internal friction of the upper hard layer would have to be investigated.

It is possible that the system used in this experiment could become a three-layered system should the zone of stress reach the bottom of the bin. Therefore, it is necessary to keep the depth of the bottom layer "infinite" to prevent this from occurring. In the present experiment, the depth of the zone of significant stress remained in the soft stratum. This remolded zone was always removed prior to conducting the next test.

CHAPTER V

CONCLUSIONS

The following conclusions have been reached as a result of this experimentation and are summarized as follows:

1. For a given footing length, there exists a non-linear (concave upward) relationship between the ratio of the length of the footing to the thickness of the upper layer and the ultimate bearing capacity of the system (Figures 19, 20 and 21).[1]

2. For a given L/B ratio, there exists a straight-line relationship between the thickness of the upper layer and the ultimate bearing capacity of the soil system (Figure 22).

3. For a given upper layer thickness, there exists a non-linear (concave upward) relationship between the ratio of the length of the footing to the thickness of the upper layer and the ultimate bearing capacity of the system (Figures 3, 4, 5 and 6).[1]

4. For a given upper layer thickness, there exists a non-linear (concave upward) relationship between the ratio of the length of the footing to the length of the upper layer and the ultimate bearing capacity of the system (Figures 7, 8, 9 and 10).

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1. The L/t curves are valid for this experiment but may not be universally valid for conditions where the L/B ratios are different.

5. For a given upper layer thickness, there exists a non-linear (concave upward) relationship between the ultimate bearing capacity of the system and the footing length (Figures 11, 12, 13, and 14).

6. For a given footing length, there exists a straight-line relationship between the ultimate bearing capacity of the system and the thickness of the upper layer (Figure 23).

7. Two distinct types of failure occurred: a punching failure and a flexural failure. The type of failure appears to be dependent upon the footing length and not upon the thickness of the upper layer.

CHAPTER VI

RECOMMENDATIONS

The following are recommendations for further study in the field of two-layered systems.

1. Studies should be undertaken using a wider variation in the thickness of the upper layer.

2. Controlled tests should be undertaken using a given footing width but varying the footing length in smaller increments in order to determine the point where the system undergoes a change from a punching failure to a flexural failure. Tests varying the footing width should also be conducted.

3. The relationship between the moduli of elasticity of the two layers and the ultimate bearing capacity of the system should be studied. This can effectively be undertaken by varying the rigidity of the two layers.

4. The effect of the cohesion and angle of internal friction of the upper hard layer on the ultimate bearing capacity of the system should be studied.

5. Full scale footing tests should be studied to see if the same type of relationships exist as with model footings.

6. The significance of edge effects on the ultimate bearing capacity of the system should be determined.

7. The effect of the contact between the upper hard layer and the lower stratum should be studied possibly by the use of strain gauges or pressure diaphragms.

APPENDIX

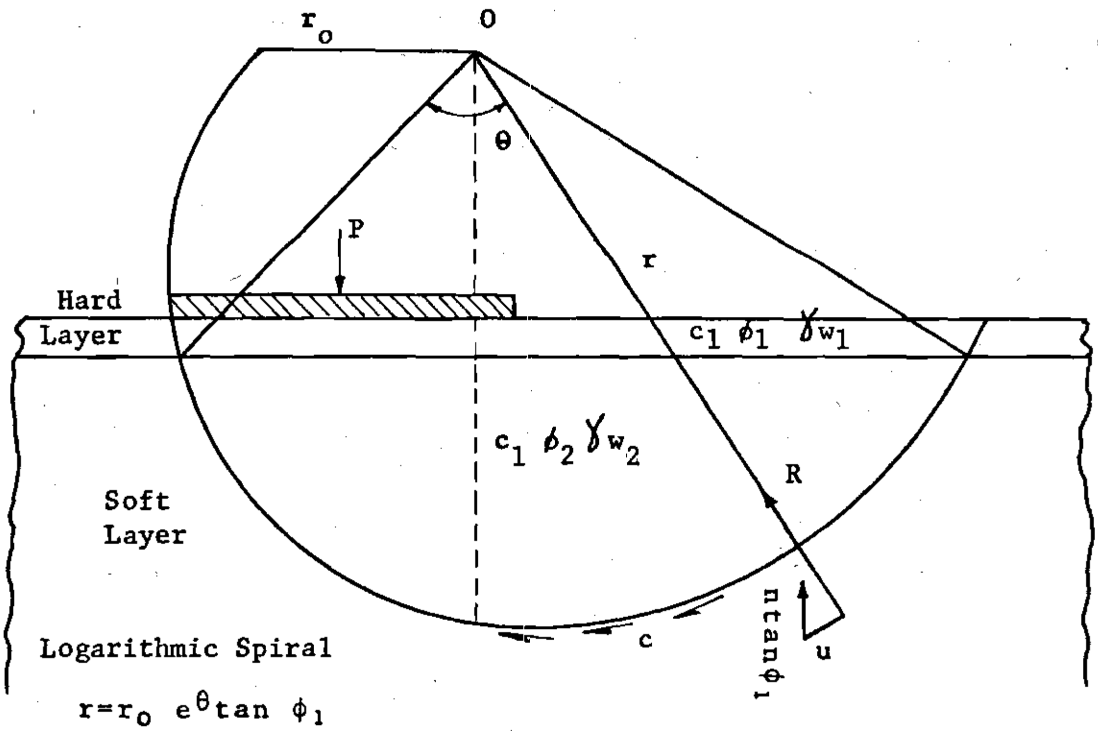

Figure 1. Logarithmic-Spiral Failure Curve

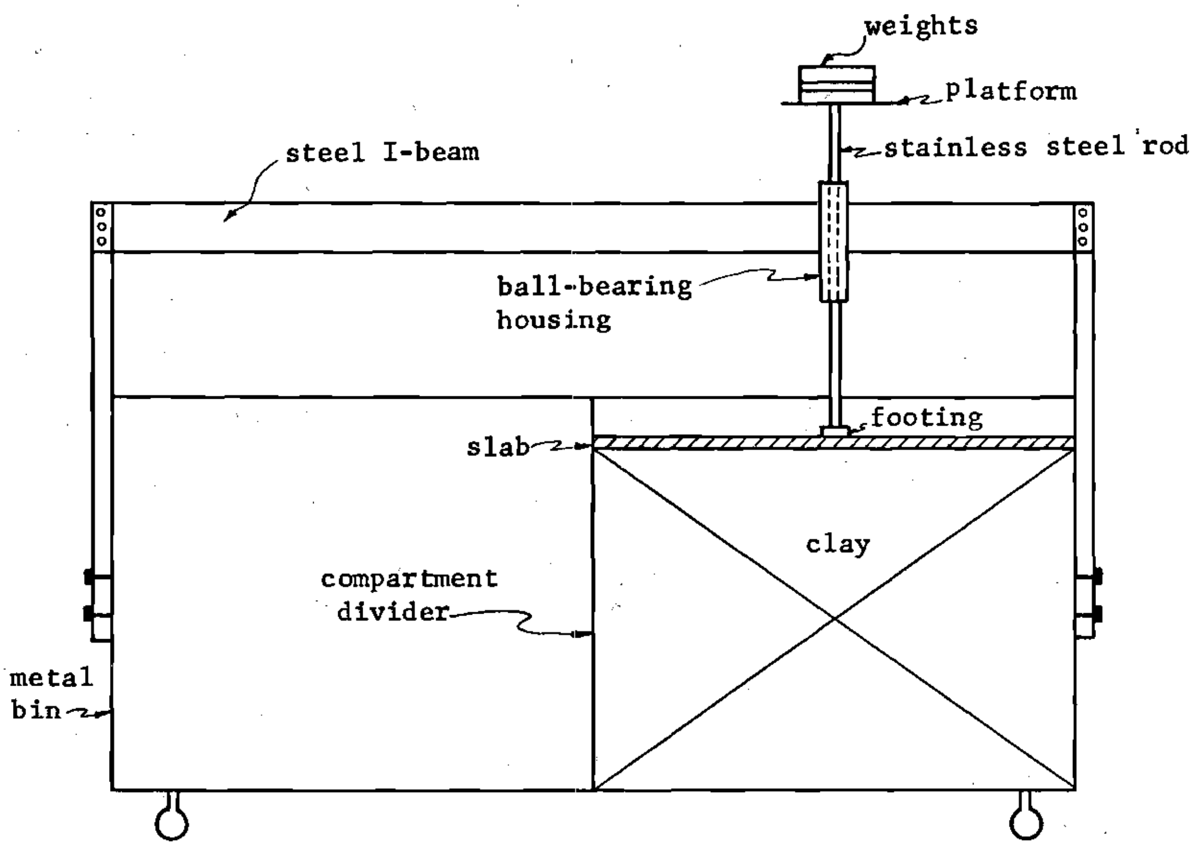

Figure 2. Load Test Set-Up

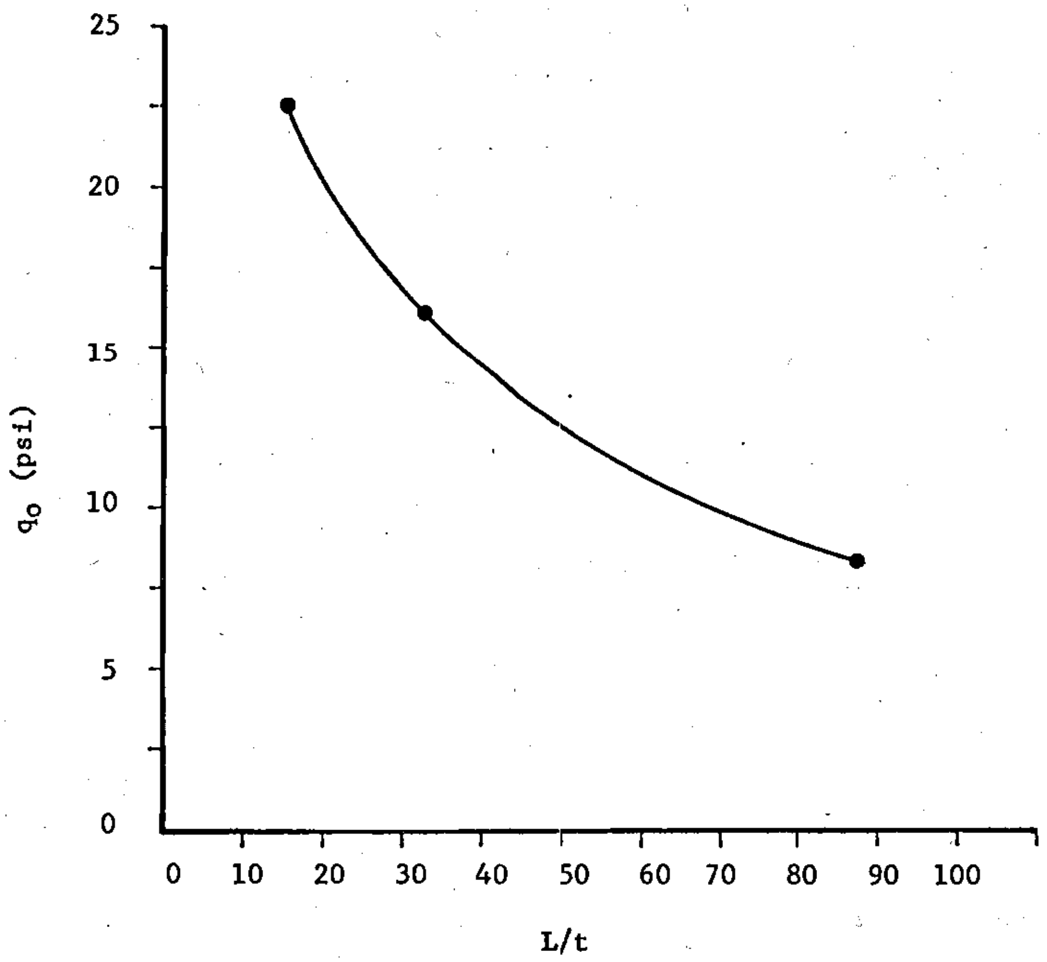

Figure 3. Ultimate Bearing Capacity vs. L/t ratio--1/4 inch slab

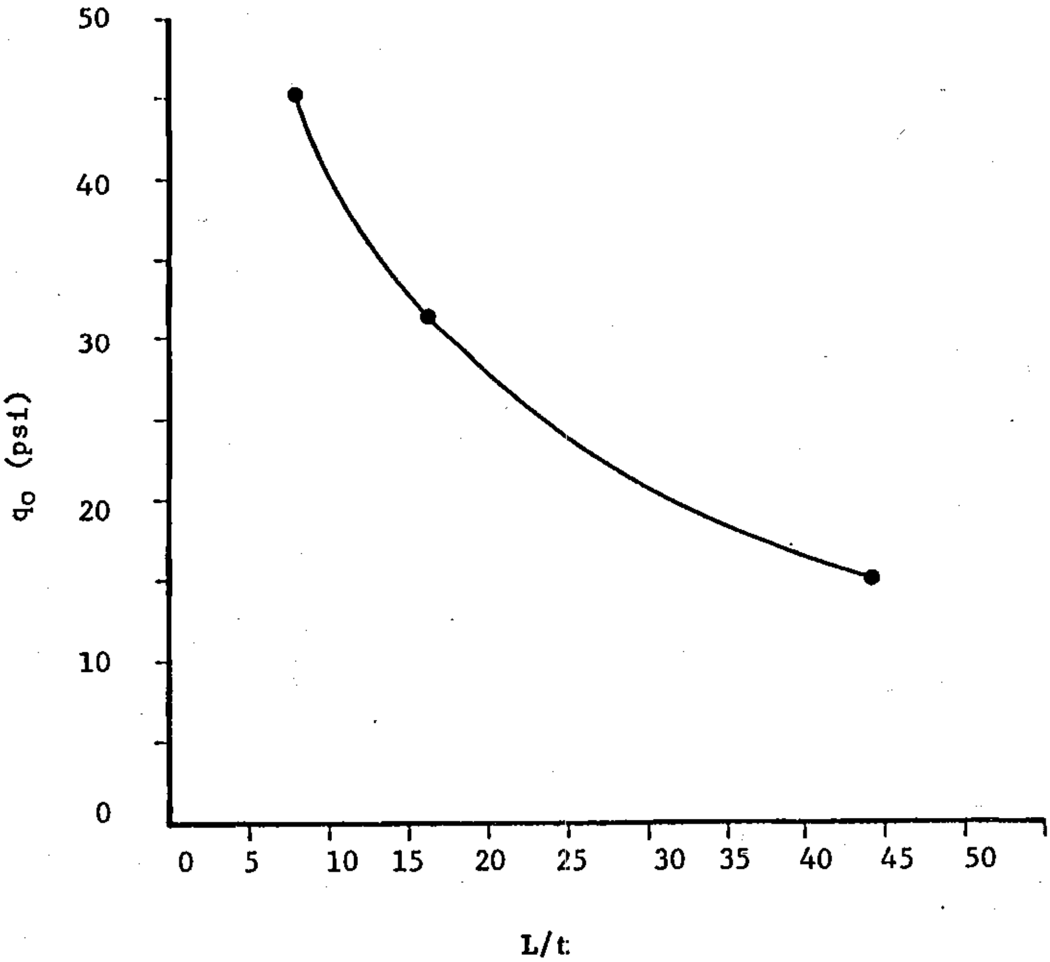

Figure 4. Ultimate Bearing Capacity vs. L/t ratio - 1/2 inch slab

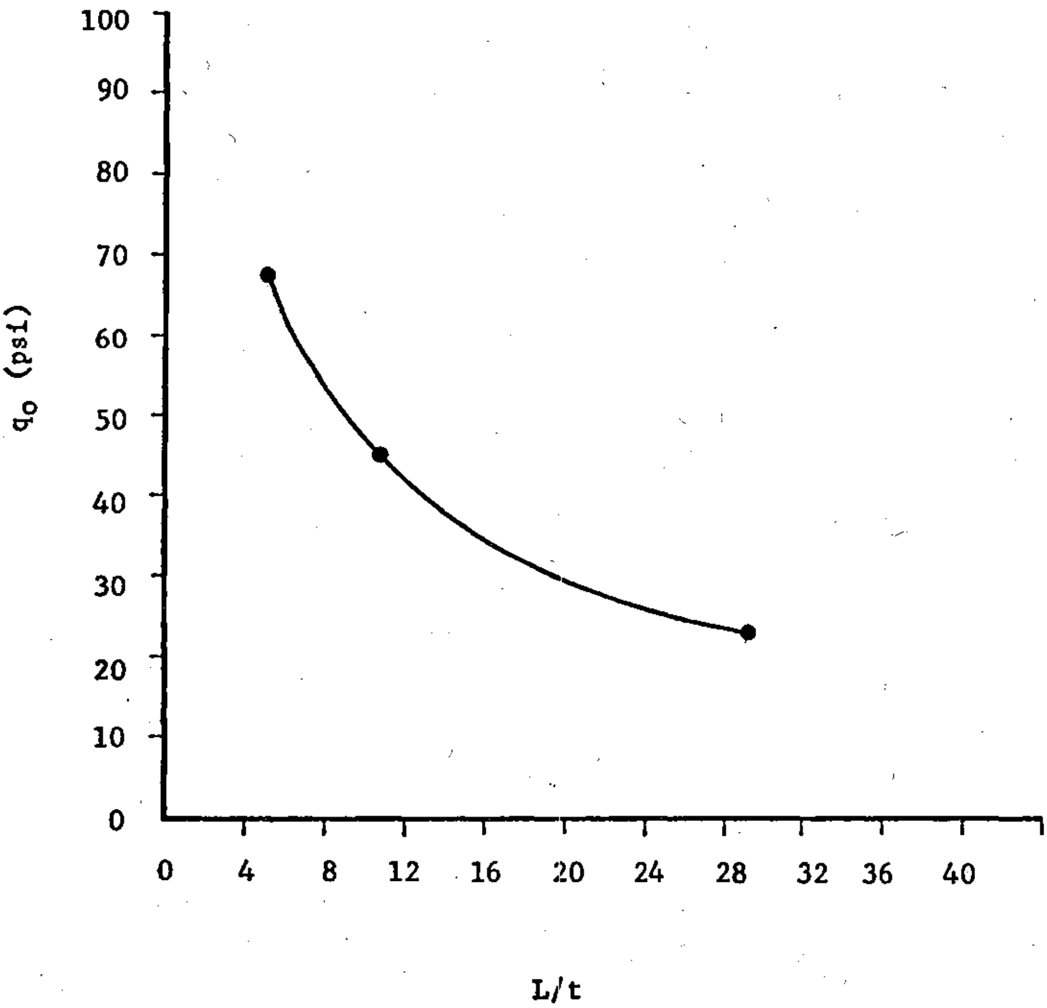

Figure 5. Ultimate Bearing Capacity vs. L/t ratio - 3/4 inch slab

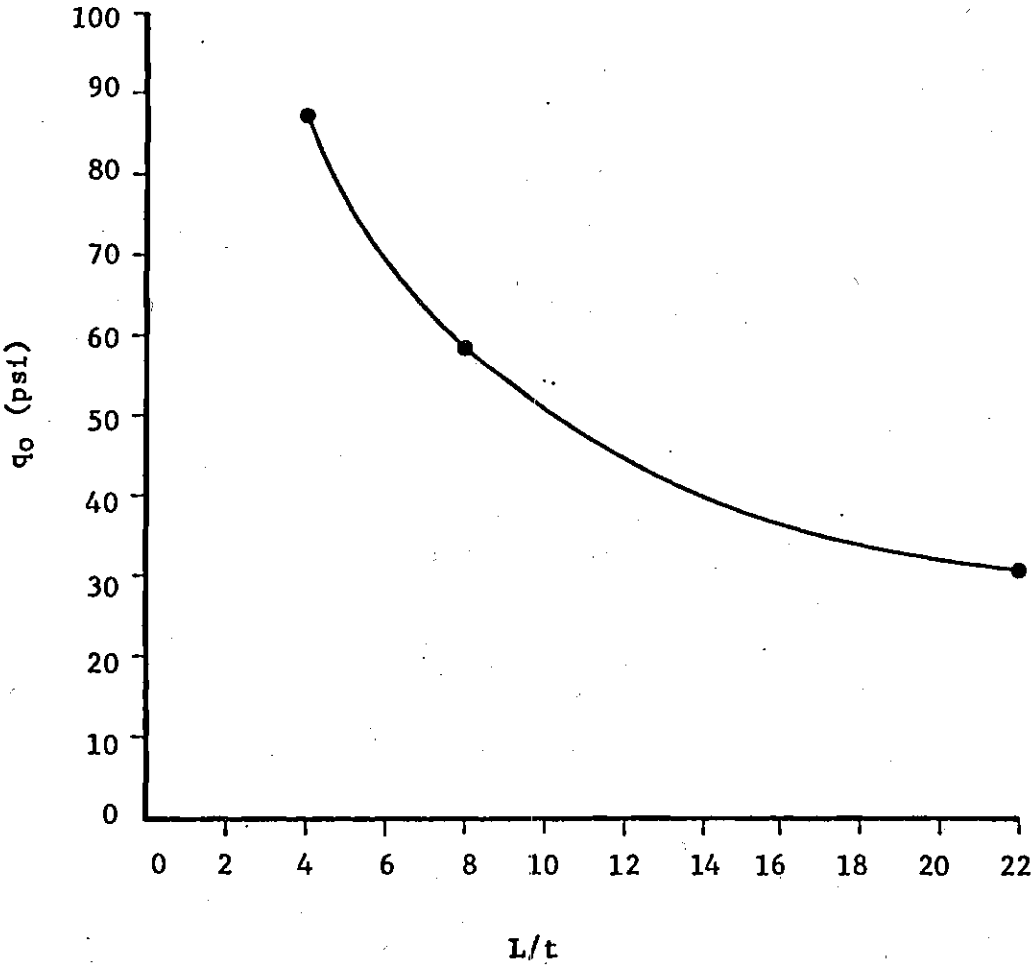

Figure 6. Ultimate Bearing Capacity vs. L/t ratio - one inch slab

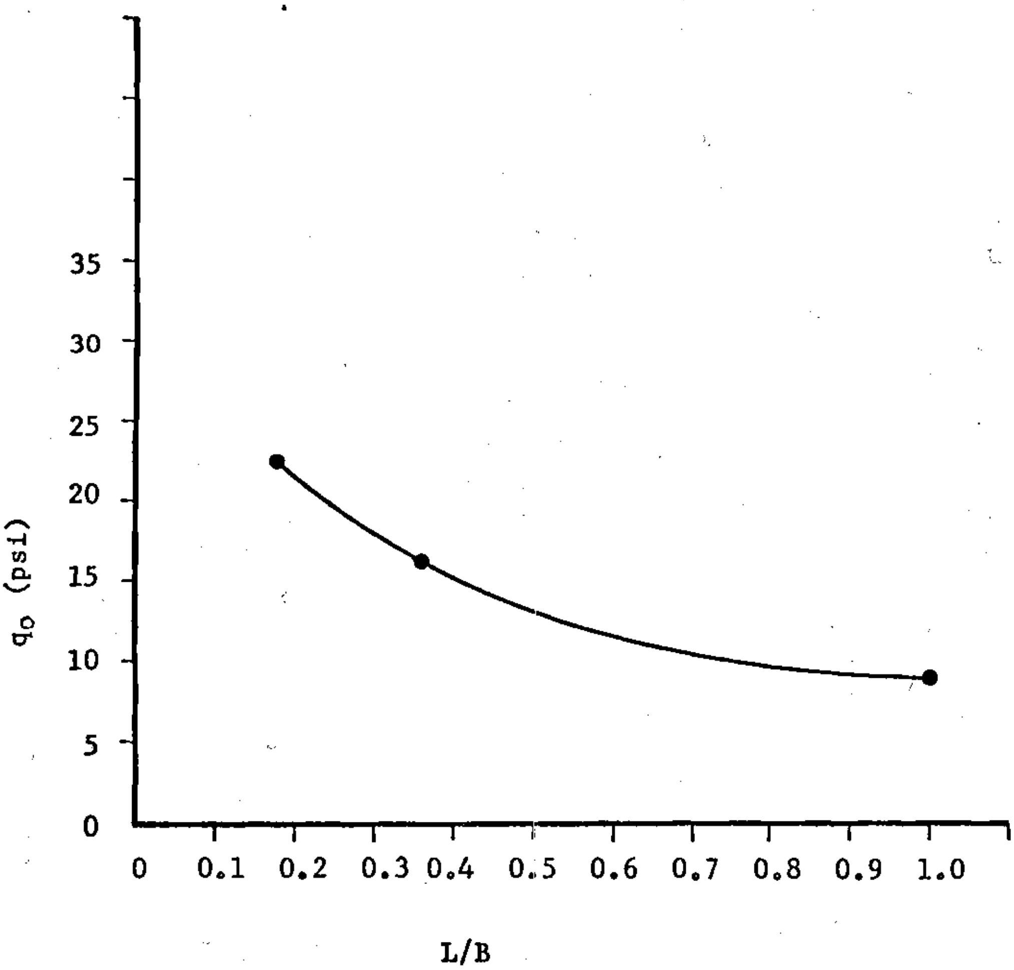

Figure 7. Ultimate Bearing Capacity vs. L/B ratio - 1/4 inch slab

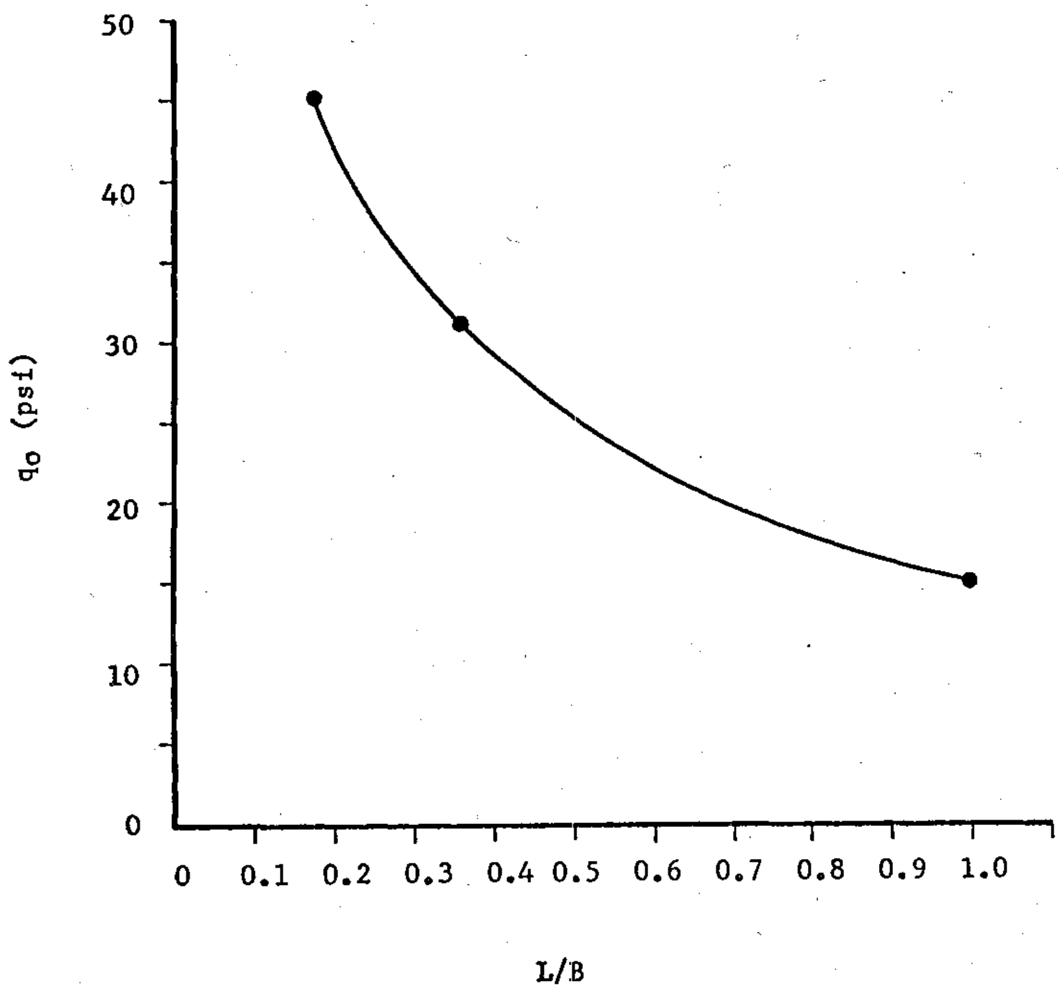

Figure 8. Ultimate Bearing Capacity vs. L/B ratio - 1/2 inch slab

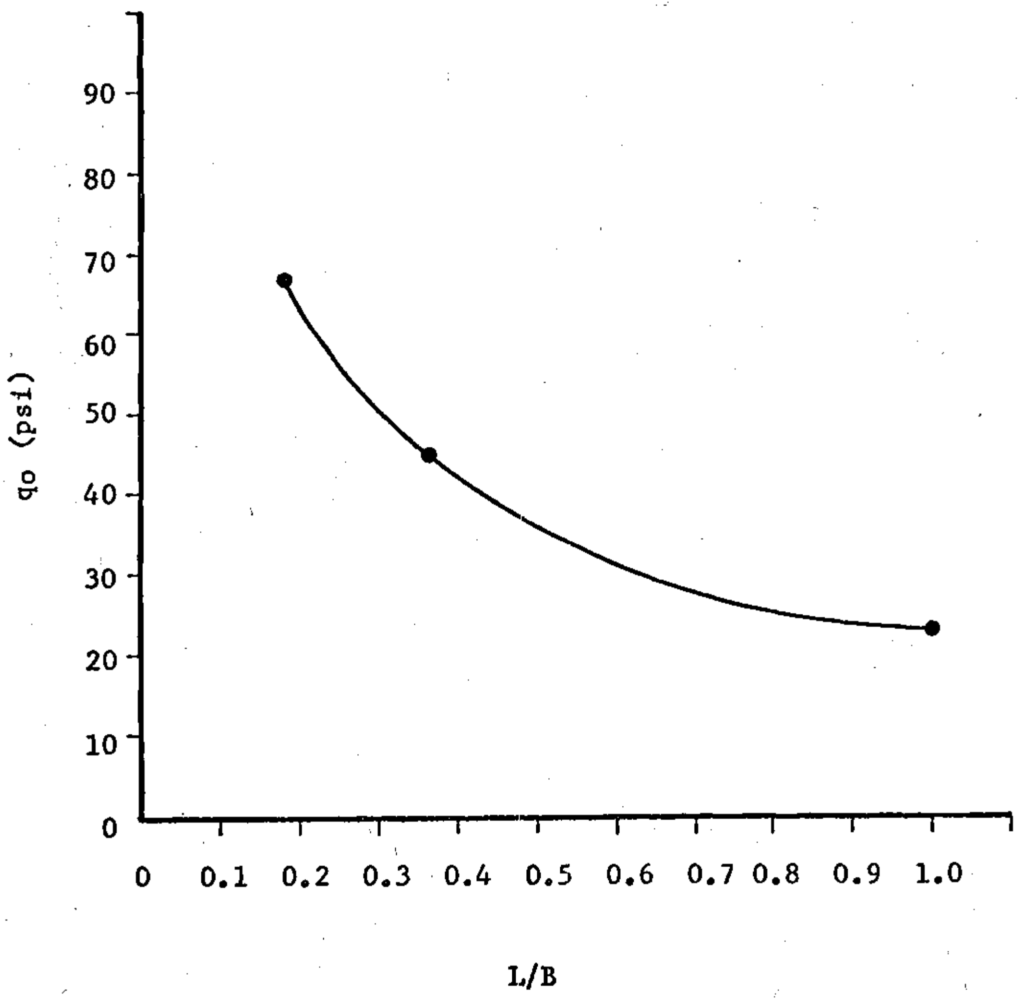

Figure 9. Ultimate Bearing Capacity vs. L/B ratio - 3/4 inch slab

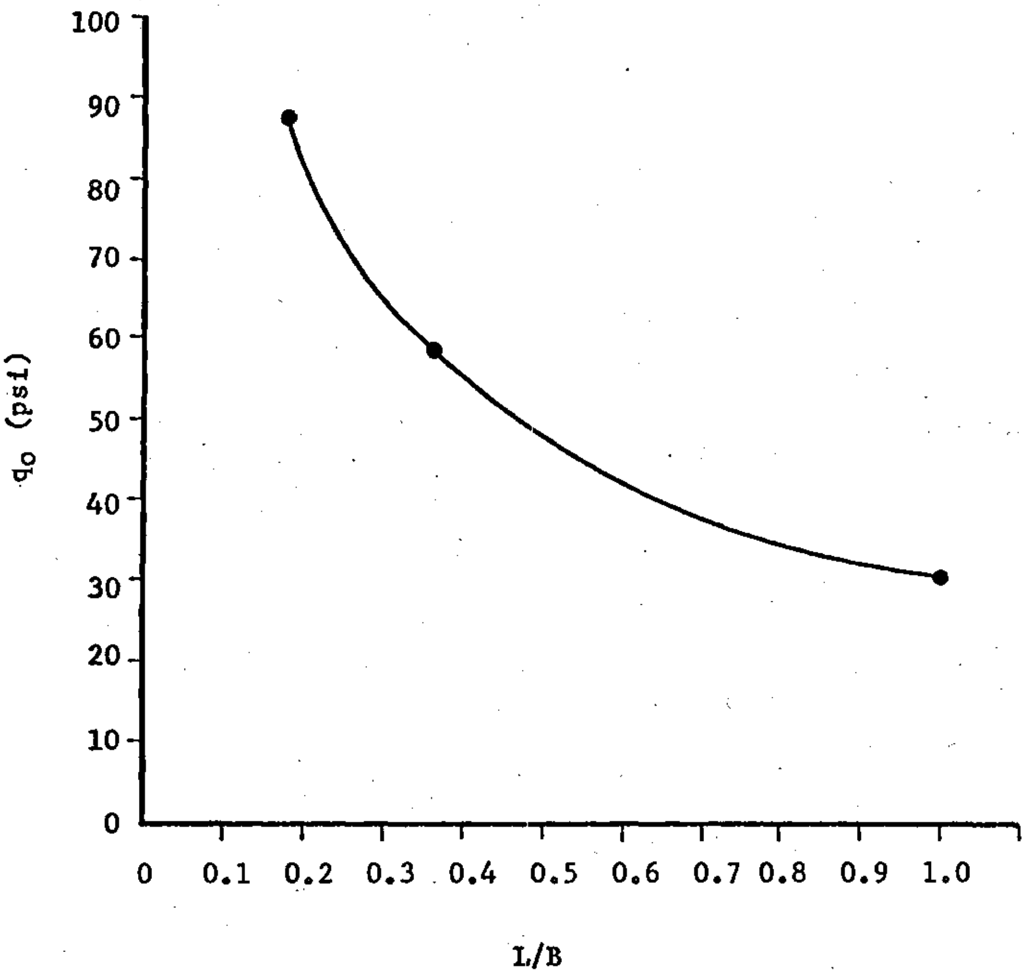

Figure 10. Ultimate Bearing Capacity vs. L/B ratio - one inch slab

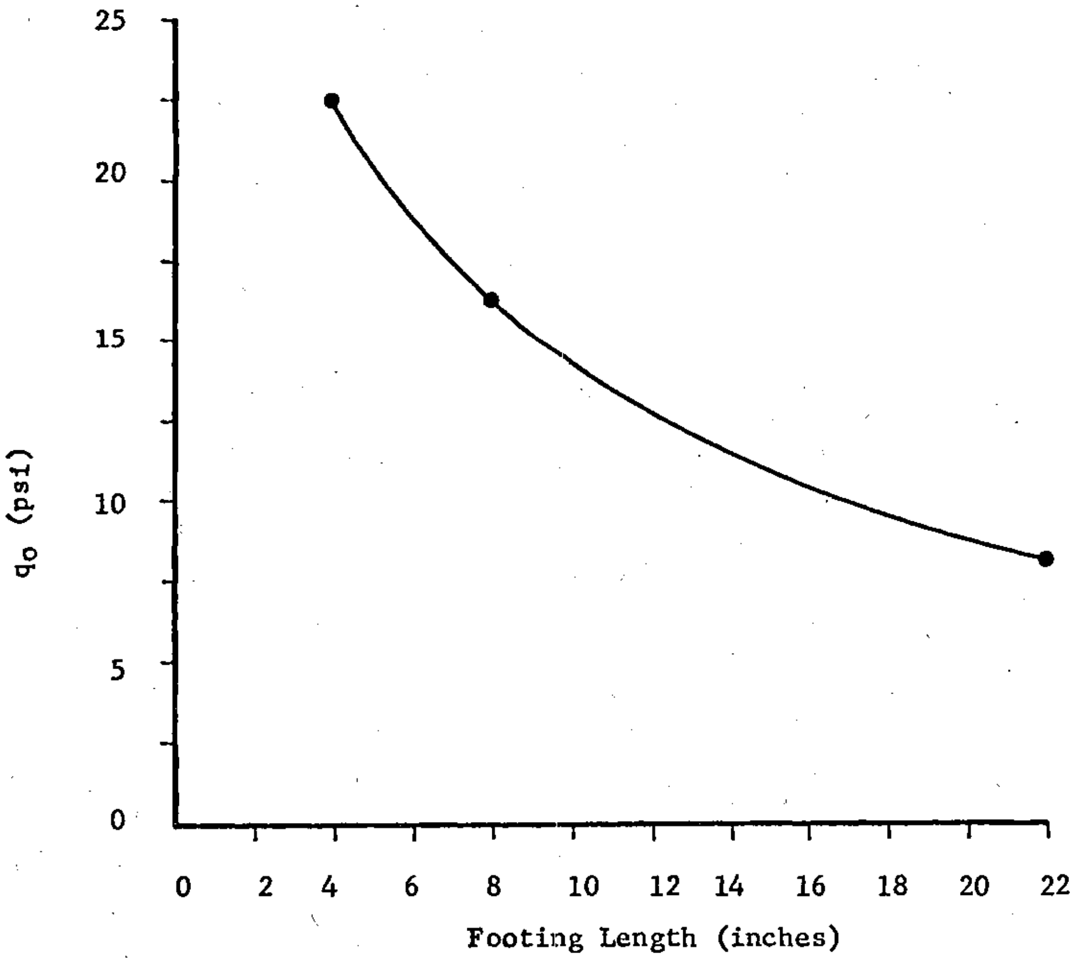

Figure 11. Ultimate Bearing Capacity vs. Footing Length - 1/4 inch slab

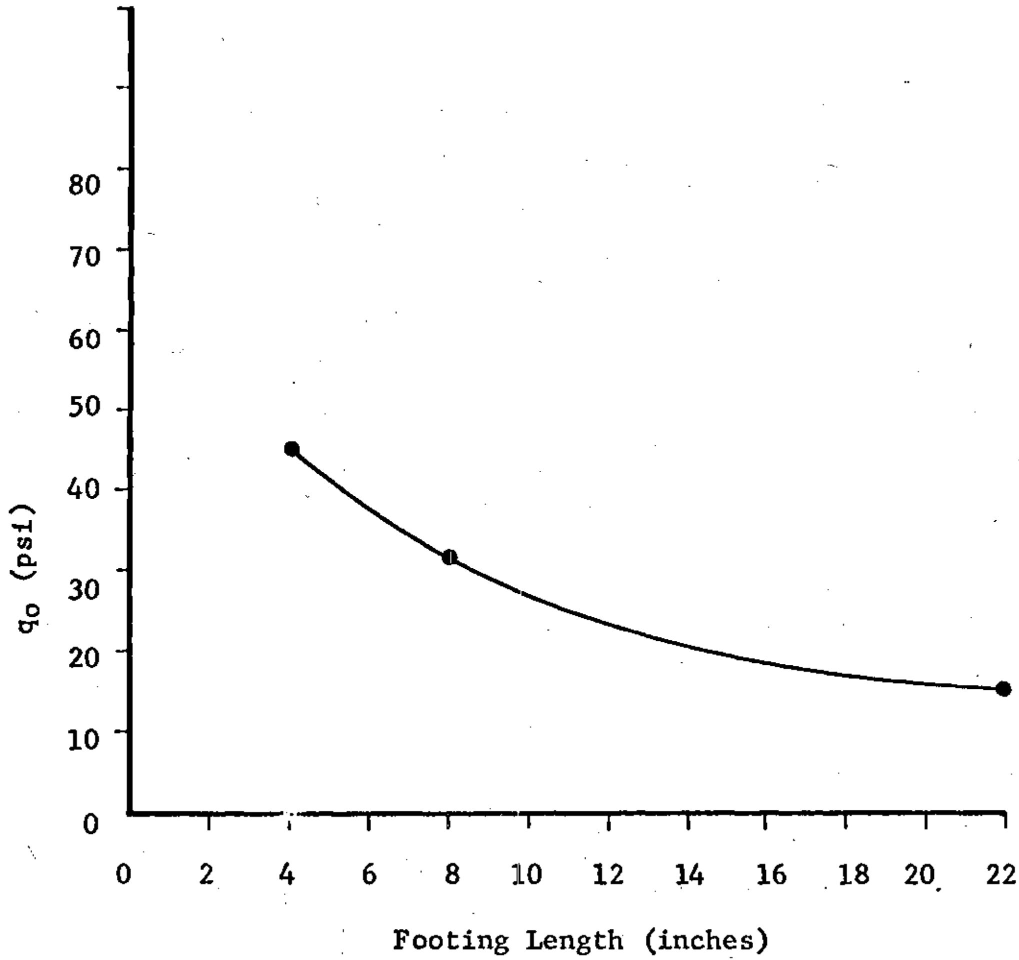

Figure 12. Ultimate Bearing Capacity vs. Footing Length - 1/2 inch slab

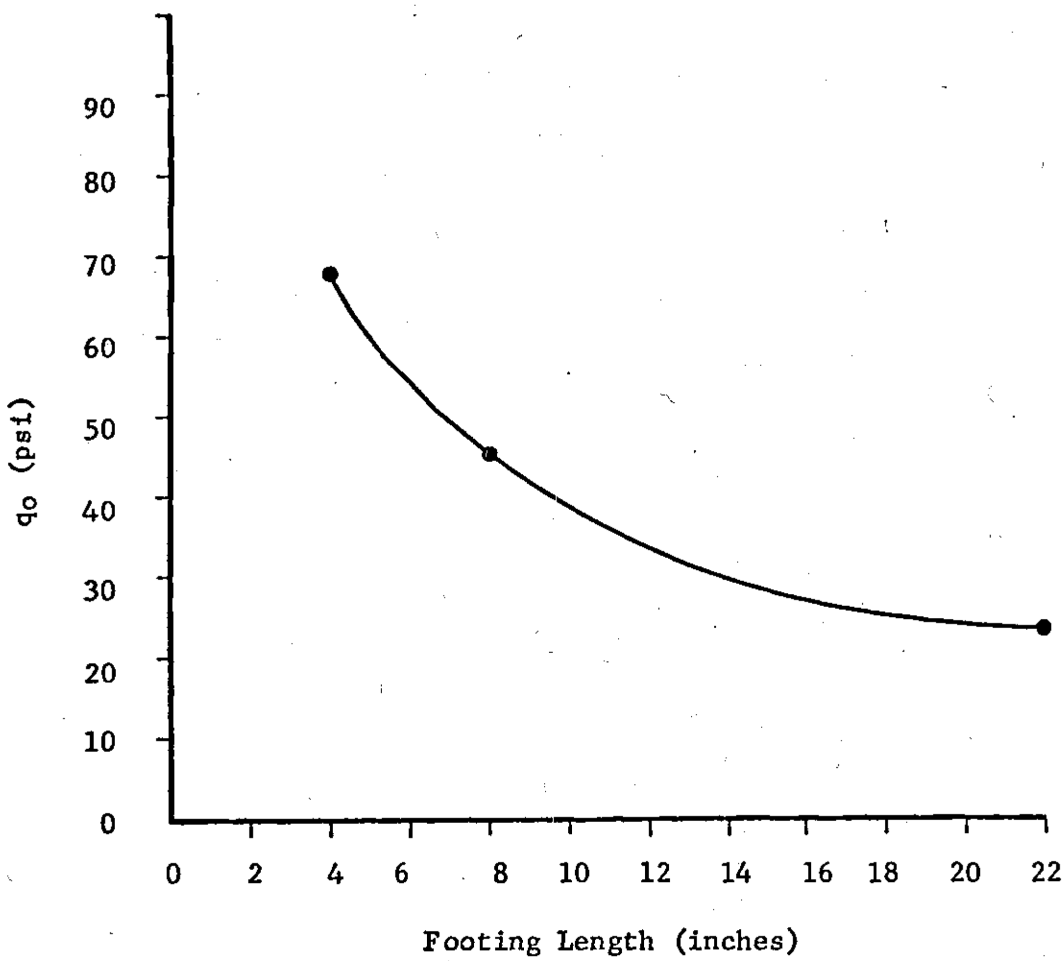

Figure 13. Ultimate Bearing Capacity vs. Footing Length - 3/4 inch slab

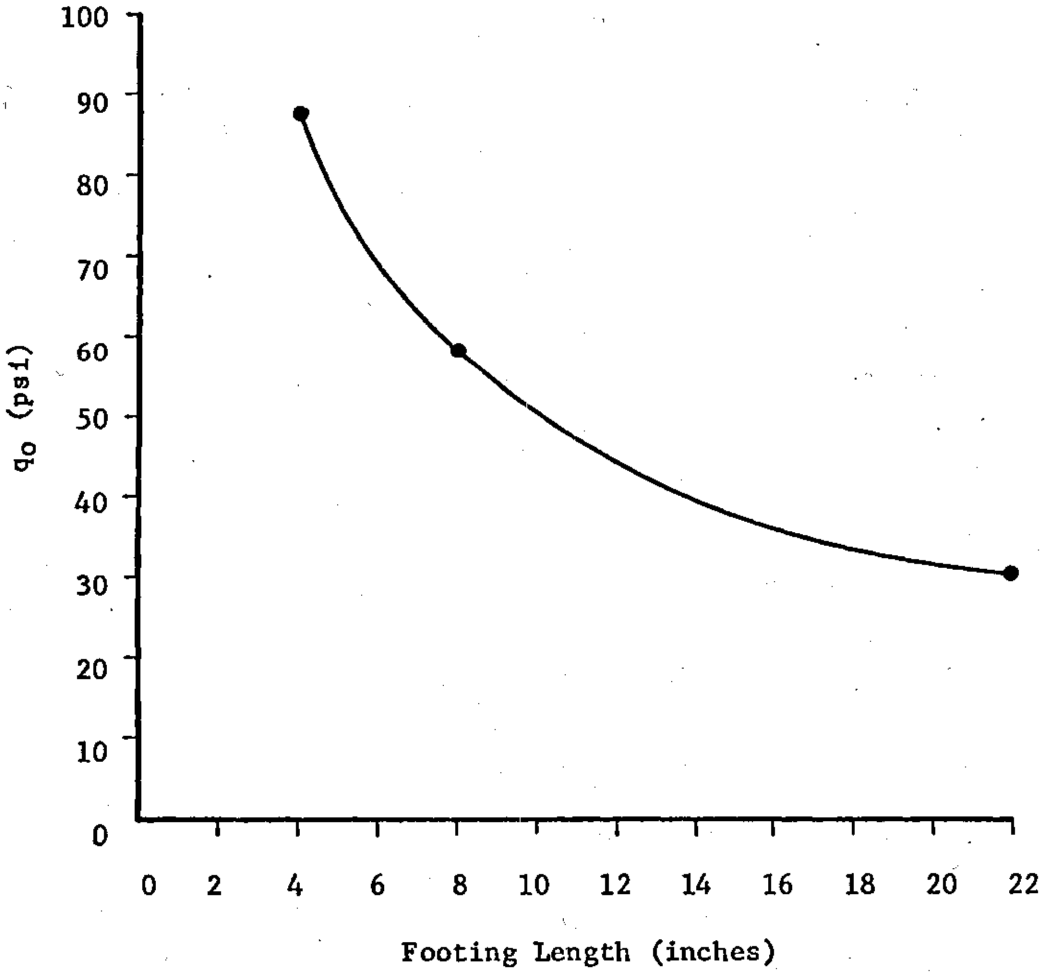

Figure 14. Ultimate Bearing Capacity vs. Footing Length - one inch slab

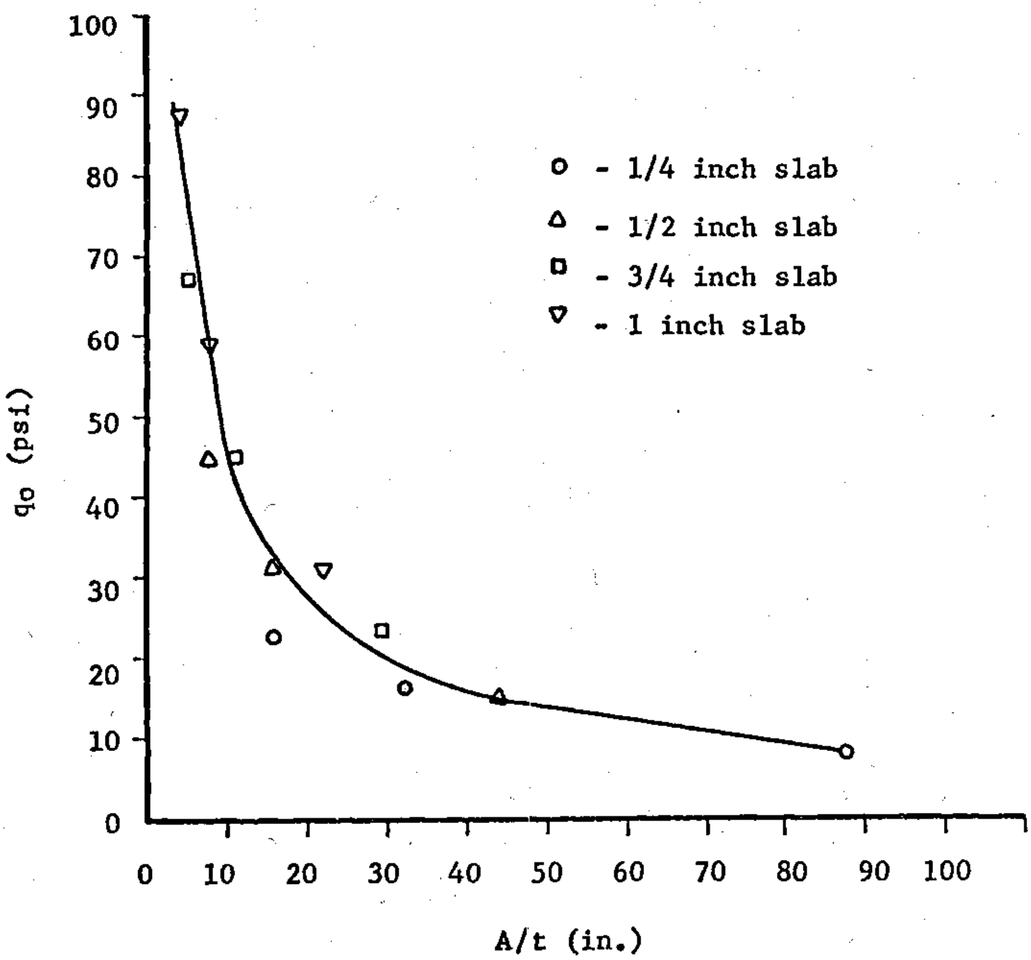

Figure 15. Ultimate Bearing Capacity vs. A/t ratio

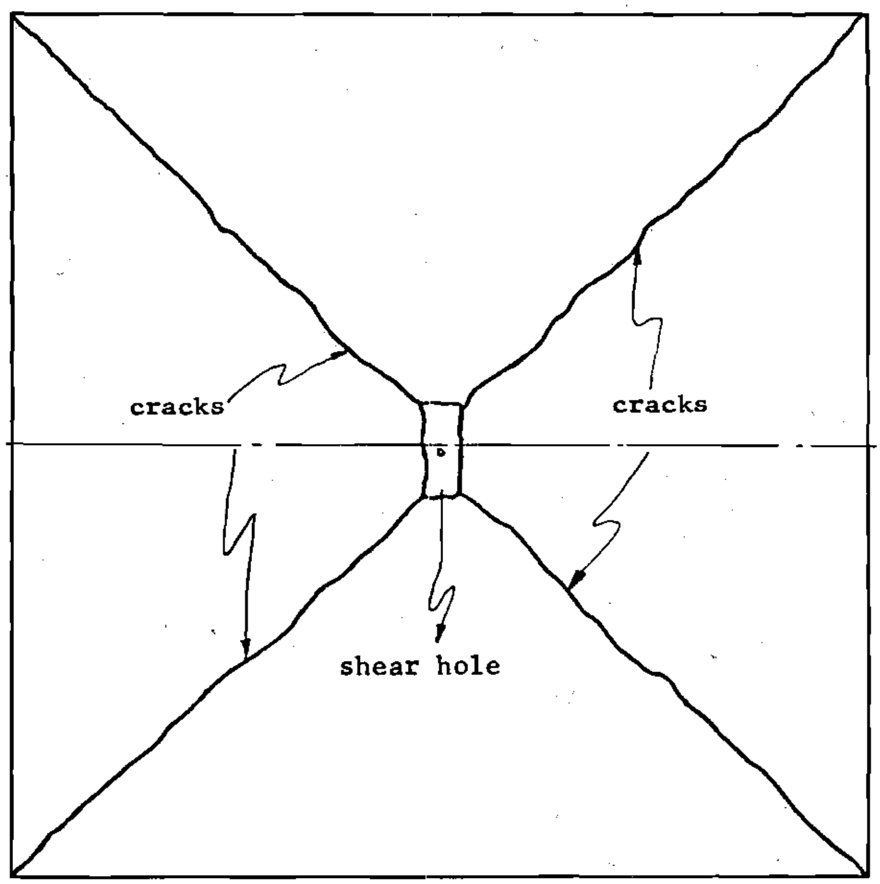

Figure 16. Crack Pattern on Top of Hard Layer for the four and eight-inch footings--all slab thicknesses.

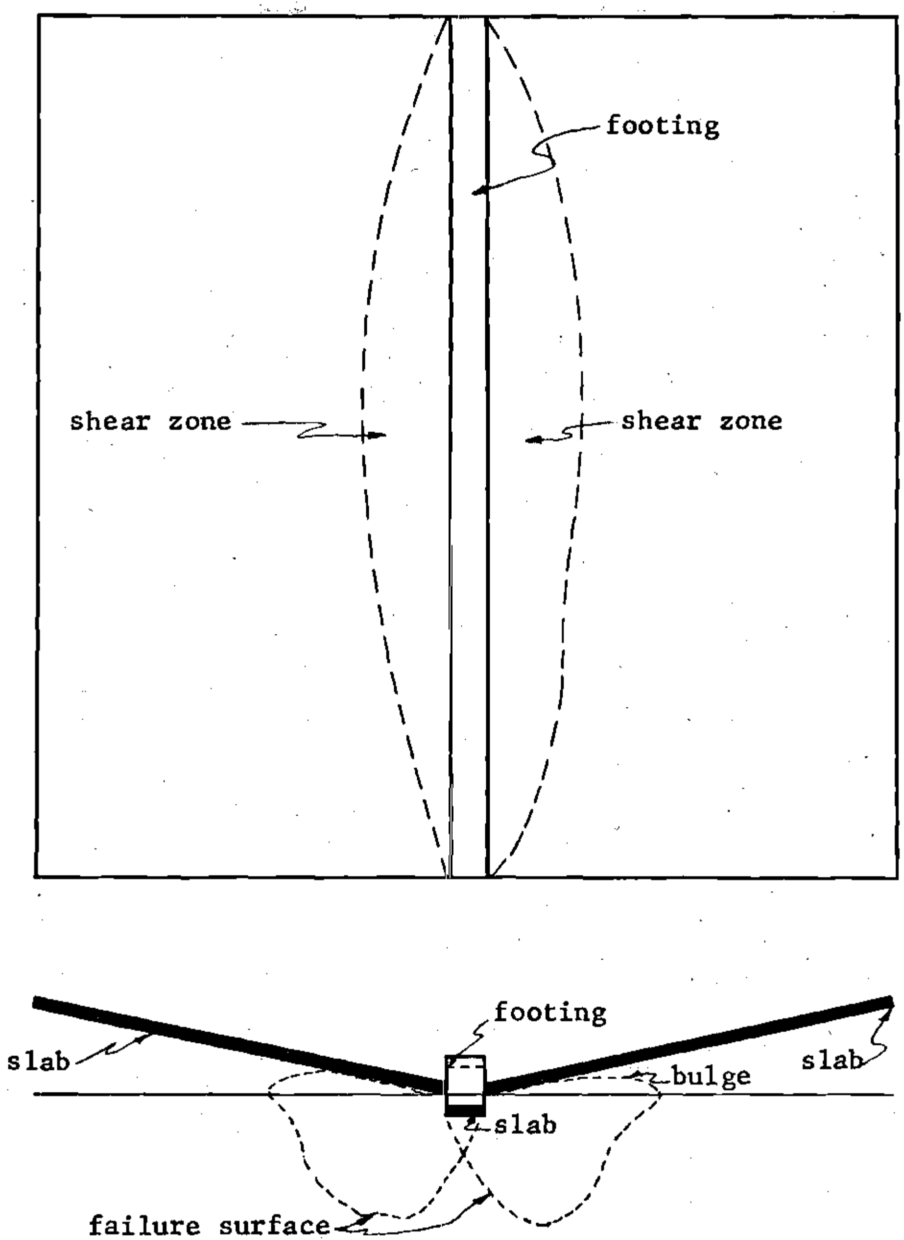

Figure 17. Crack Pattern on Top of Hard Layer for the 22 inch footing--most slabs.

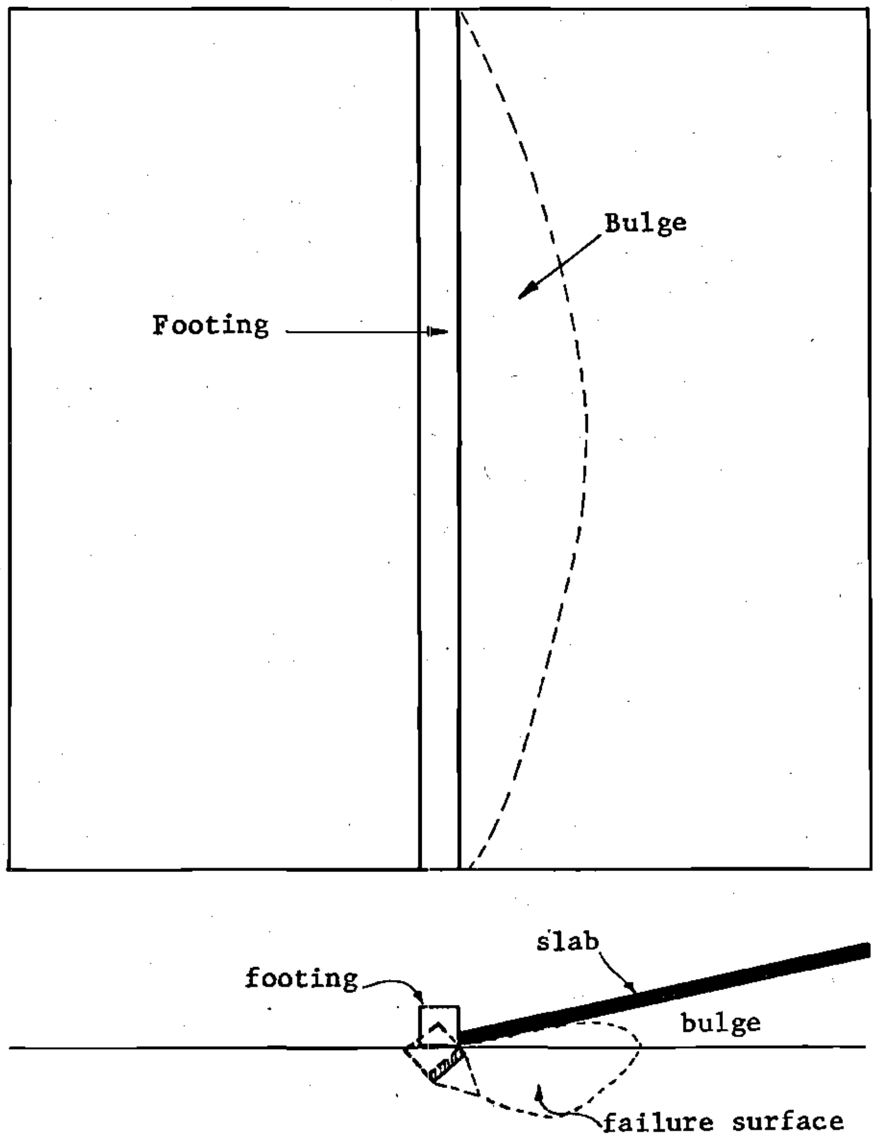

Figure 18. Crack Pattern on Top of Hard Layer for the 22 inch footing--some slabs.

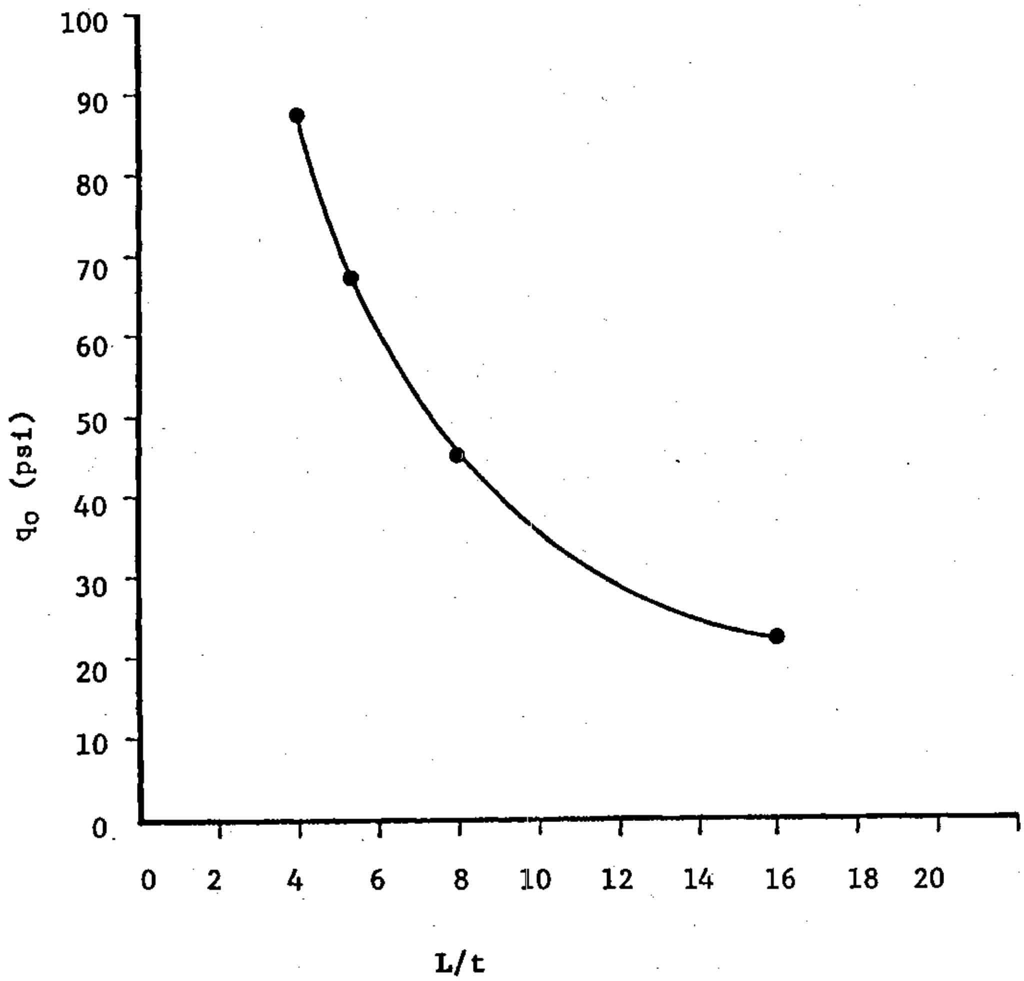

Figure 19. Ultimate Bearing Capacity vs. L/t ratio--four inch footing

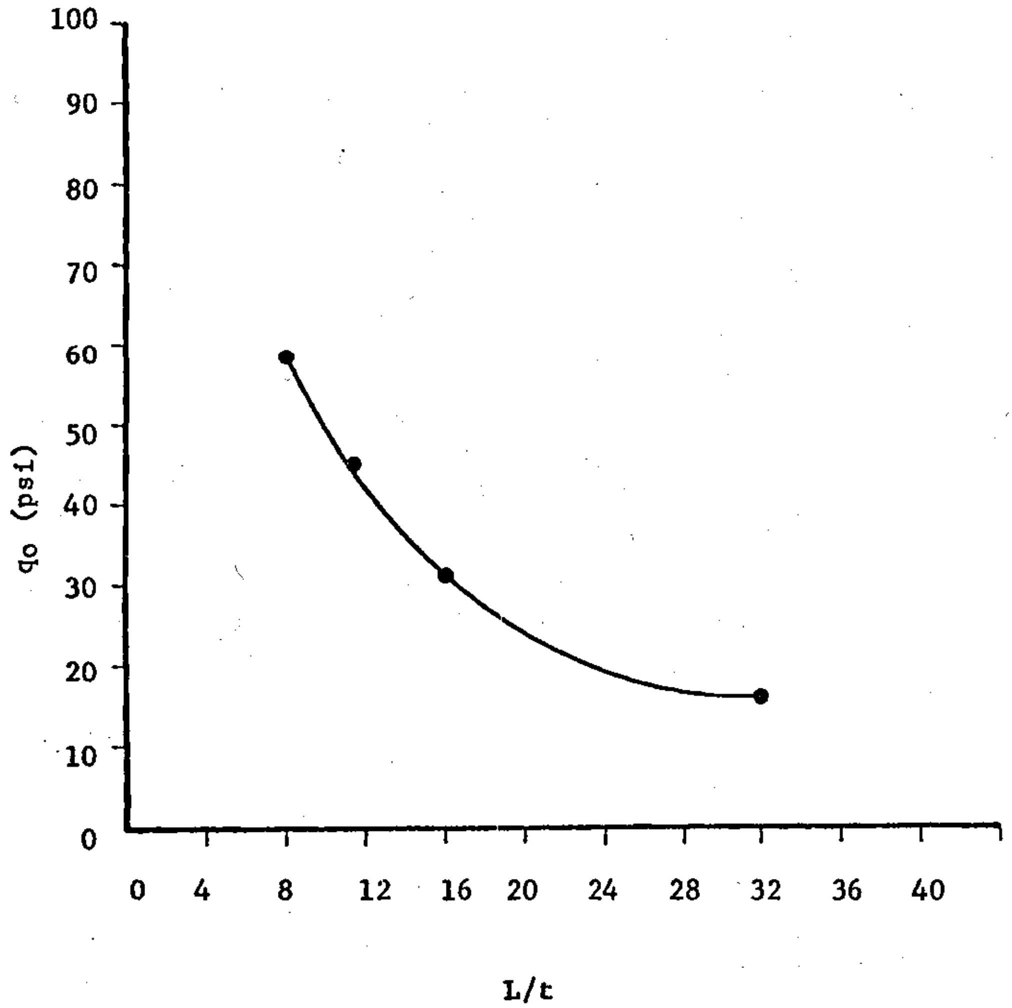

Figure 20. Ultimate Bearing Capacity vs. L/t ratio - eight inch footing

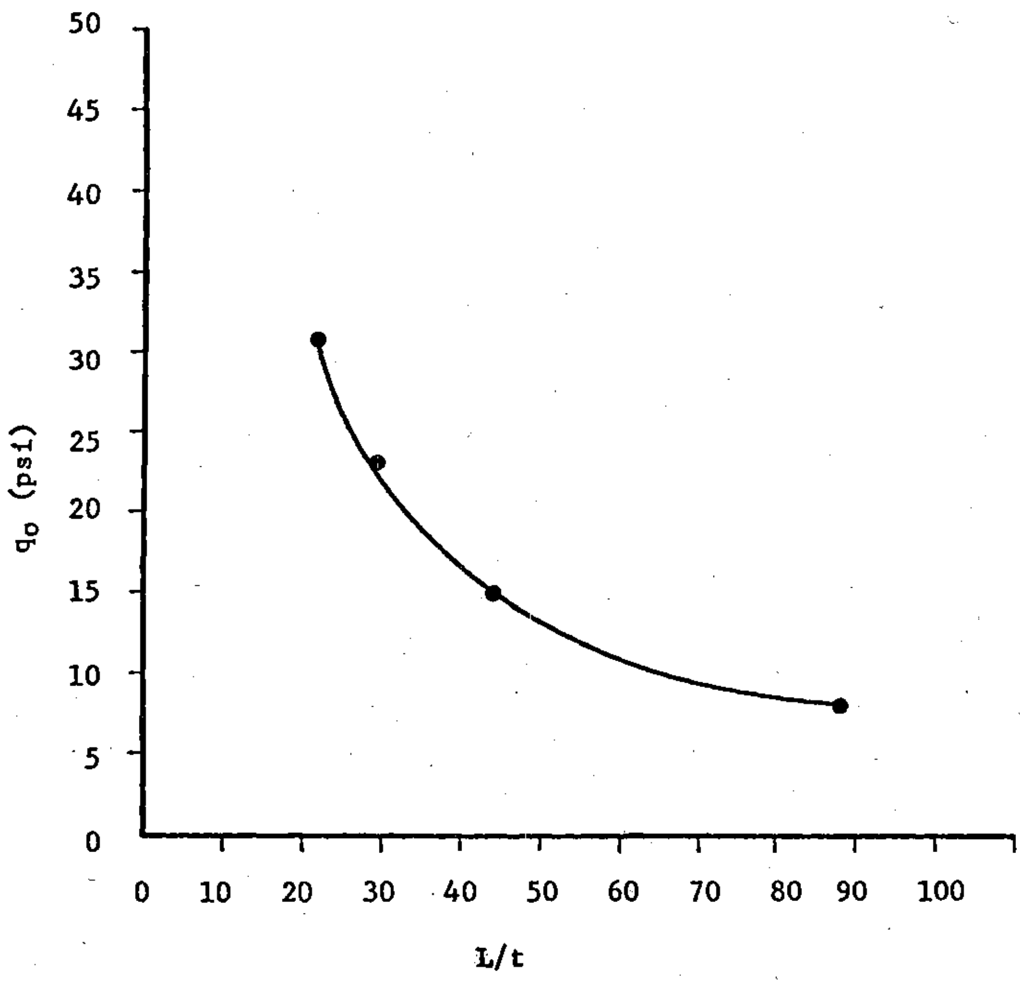

Figure 21. Ultimate Bearing Capacity vs. L/t ratio - 22 inch footing

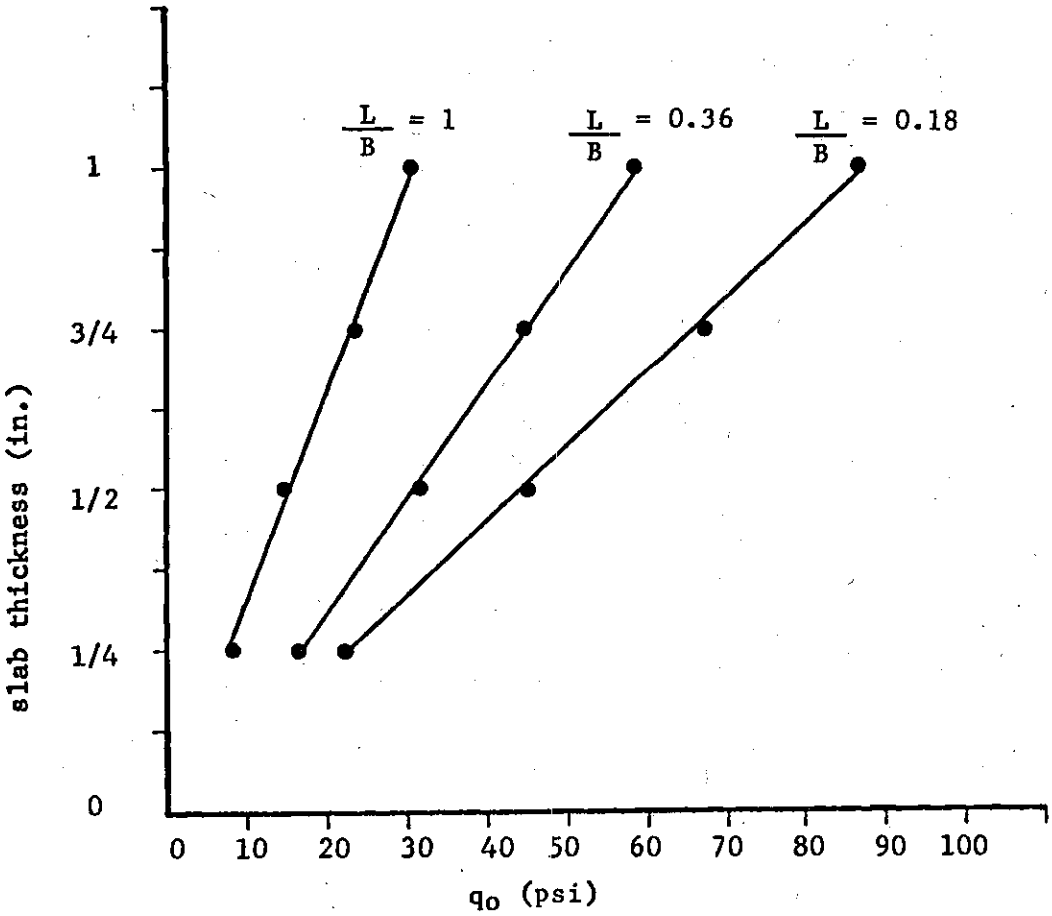

Figure 22. Ultimate Bearing Capacity vs. Slab Thickness for Each L/B Ratio.

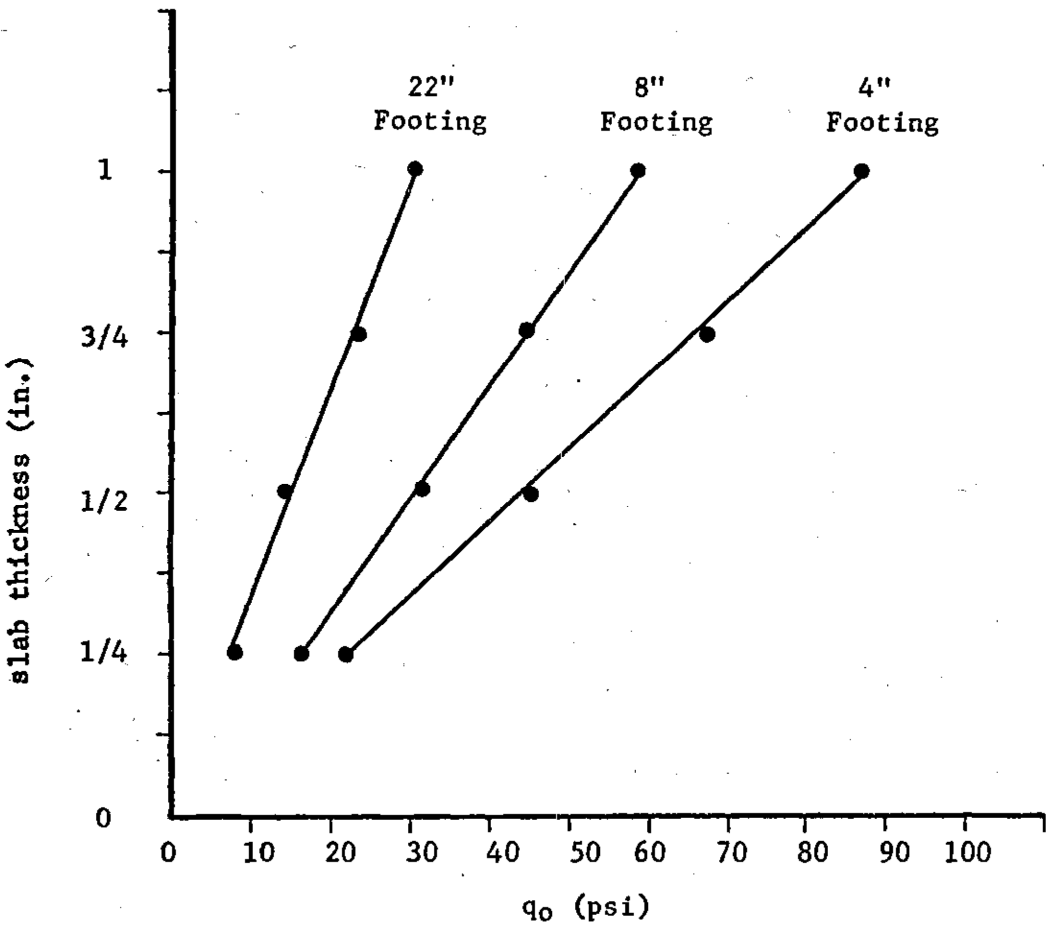

Figure 23. Ultimate Bearing Capacity vs. Slab Thickness for Each Footing Length.

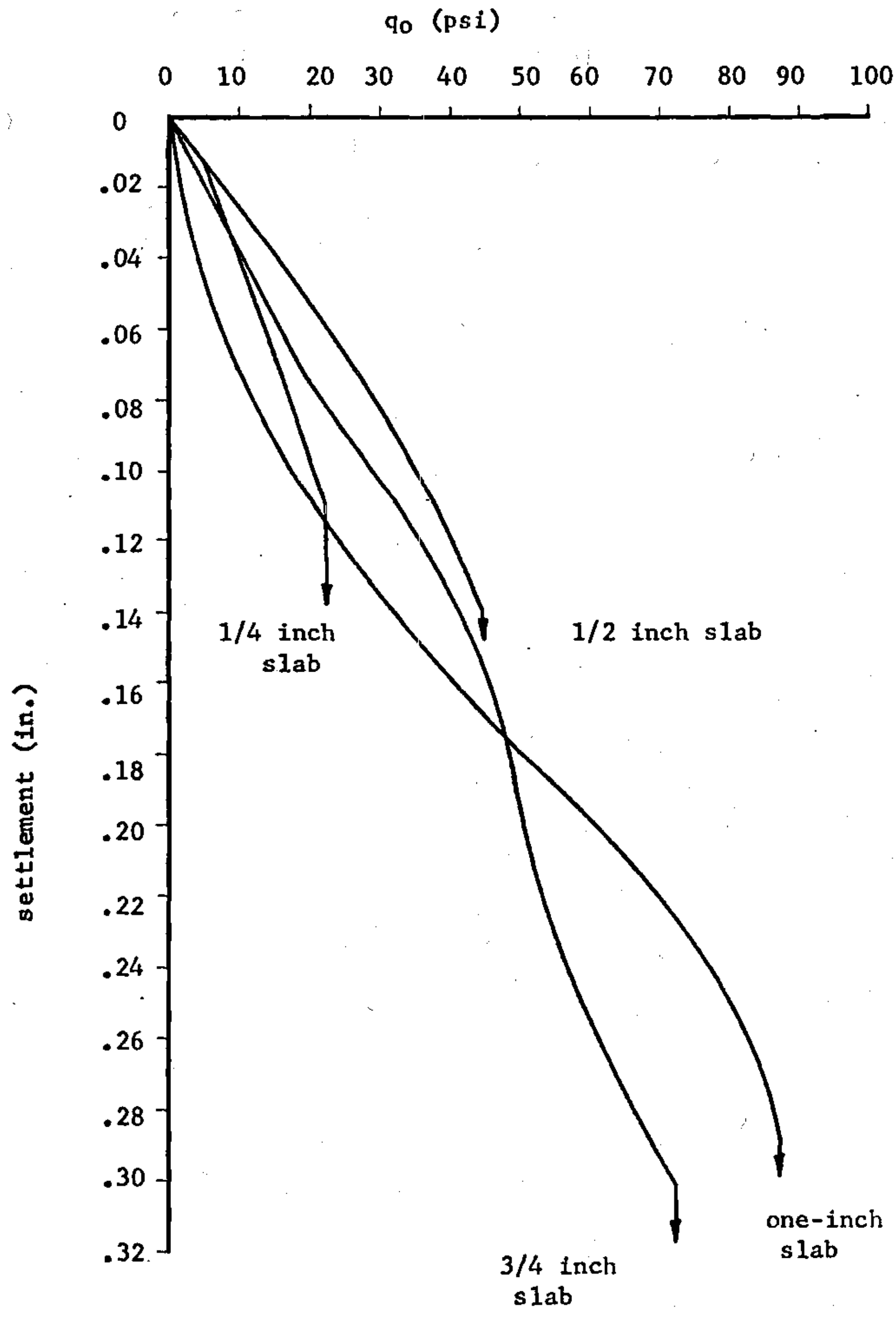

Figure 24. Pressure-Settlement Curves for Each Slab Thickness - four inch footing.

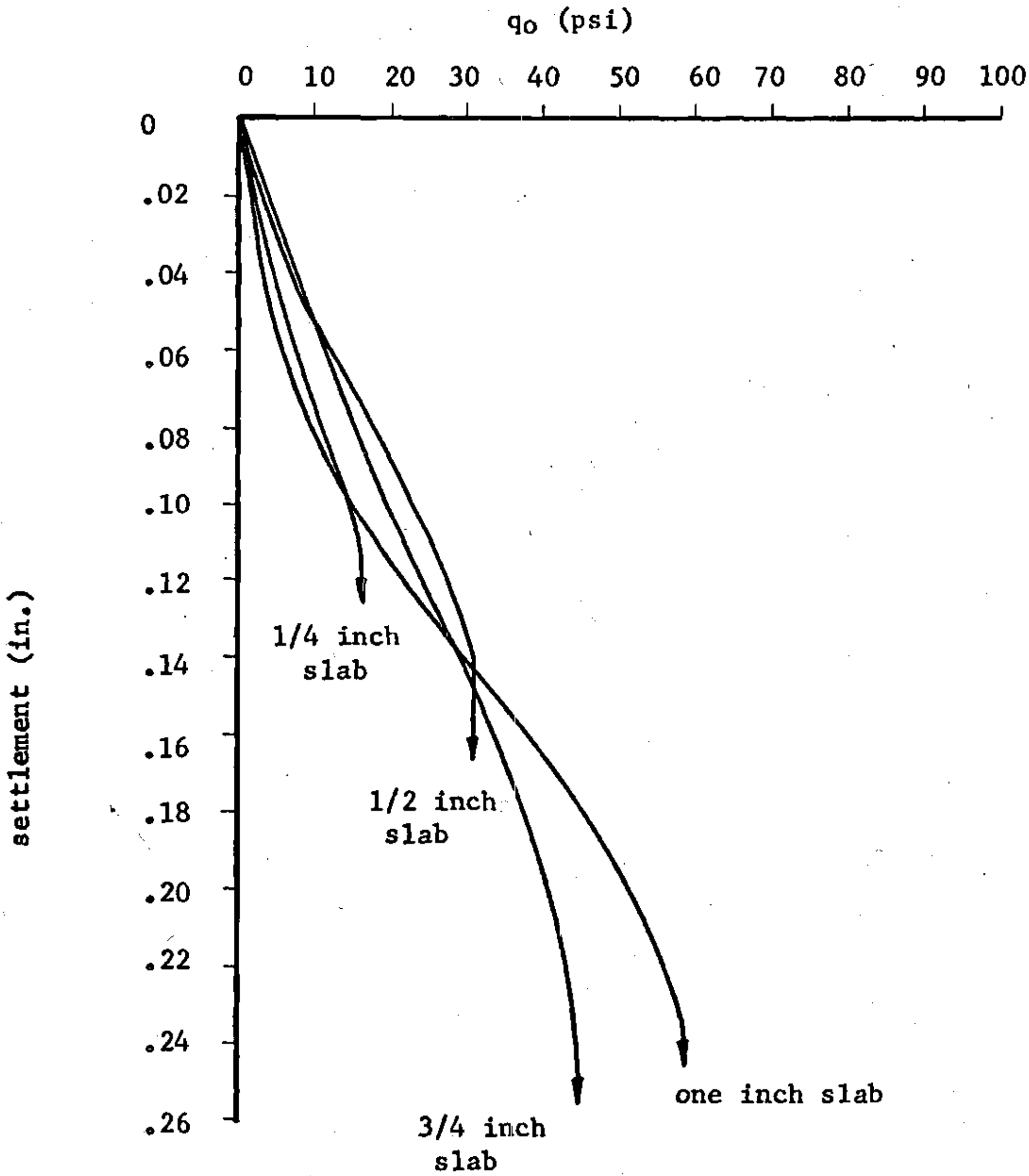

Figure 25. Pressure-Settlement Curves for each Slab Thickness - 8 inch footing

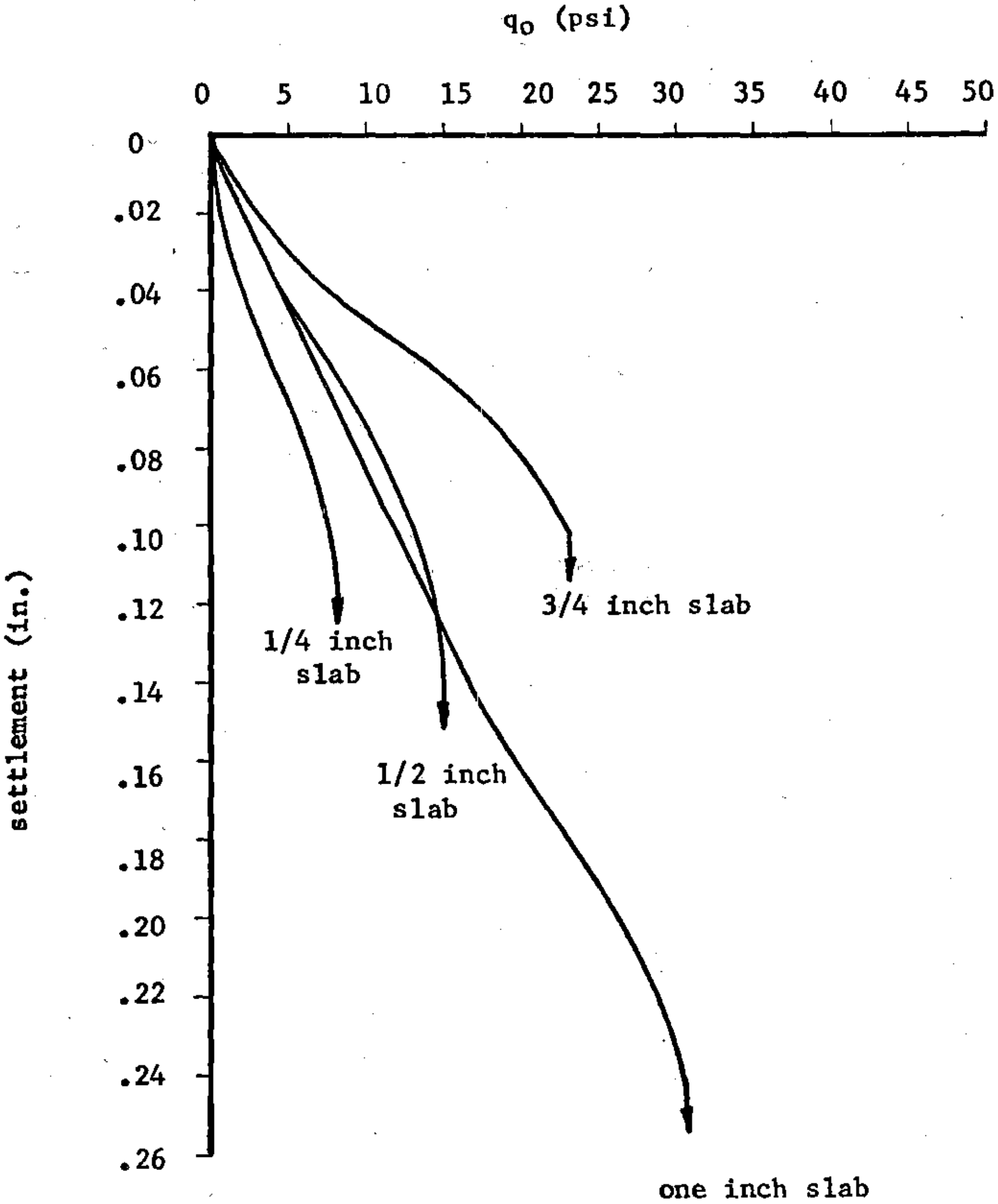

Figure 26. Pressure-Settlement Curves for each Slab Thickness - 22 inch footing

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